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# LONG-SPAN RAILWAY BRIDGES

(*REVISED EDITION*);

AND

# SHORT-SPAN RAILWAY BRIDGES.

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[illegible] TICAL
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# [illegible] RAILWAY BRIDGES.

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*In hoc genere, duo vitia vitanda sunt: unum, ne incognita pro cognitis habeamus, hisque temerè assentiamur. Alterum est vitium, quòdquidam nimis magnum studium multamque operam in res obscuras atque difficiles conferunt, eâsdemque non nècessarias.*—Cic. de Off., lib. i., cap. 6.

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# PREFACE.

The portion of the following work treating on Long-span Railway Bridges, was first published some seven years ago, so but little introduction will now be required. As stated in the Preface to the First Edition, on proceeding with the investigation, it was at once seen that a strictly mathematical treatment of the subject would entail lengthy and involved formulæ, and absorb far greater space than was available for the purpose. Accordingly, the various hypotheses, which it is absolutely necessary to make in an inquiry of this nature, are framed as comprehensively as possible; and in many instances the result of a careful balancing of probabilities is given without exhibiting the process by which it has been evolved. In short, elimination, and not elaboration, has been the aim throughout.

The succeeding portion of the work, referring to Short-span Railway Bridges, is now published for the first time. The aim of the Author has been to maintain unity of design, so far as possible, in the treatment of the two branches of his subject, but the altered con-

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ditions have to some extent suggested different modes of procedure. Thus, formulæ, numerous and indispensable in the former division of railway bridges, will be conspicuous by their absence in the present division. Nor could they be advantageously introduced, for the data necessary to obtain the constants in any general formula for the weight of iron in Short-span Railway Bridges, will suffice for the immediate compilation of tables of weights, without the intervention of a formula.

The prominent position assumed by formulæ in the investigations referring to the first group of railway bridges, is, in the second, usurped by considerations bearing upon the special action of a rolling load. In a long-span bridge, but little speciality attaches to a rolling load; in a short-span bridge it is the point above all others demanding the most anxious consideration from the engineer. The Author's experience for some years past, as to the effects of heavy rolling loads upon the most severely worked line in the world,—where the bridge girders are bent by passing wheels as many times in a few hours as upon an ordinary line they would be in a year,—has enforced upon him the conclusion, that the destructive action of a frequently recurring rolling load is habitually under-rated; and that, as a consequence, in many instances heavy works for maintenance will have to be incurred in the future.

The ordinary practice of lumping together dead and rolling loads, and adopting an uniform working strain of 5 tons per square inch, whatever the respective proportions of the said loads may be, is totally indefensible, and implies an unconscious exhibition, on the part of the designer, of timidity in some instances, of temerity in others.

If the following pages contribute at all to the development of a more rational practice in the above respect, and to the adoption of a more uniform standard for the strength and weight of railway bridges than at present obtains, the purpose of the Author will have been achieved.

B. B.

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# LONG-SPAN RAILWAY BRIDGES.

A BRIDGE, as defined by our early lexicographers, is "a structure to carry a road across a watercourse;" and although this interpretation of the word is not sufficiently comprehensive to include all cases in the present advanced stage of the art of building, yet, if we limit the application of the term to *long* spans alone, we may even render it still more explicit, and with very little liability to error define a long-span bridge to be a structure for carrying a *railway* across a watercourse. The reasons why this is the case are sufficiently obvious. The condition necessitating the adoption of a long span is generally, either that a certain width of opening must be provided, clear of all obstructions, or that the expense of carrying up a number of lofty piers is, owing to some difficulty in obtaining secure foundations, so great as to render it more economical to reduce the number of individual supports, and so concentrate the resulting greater load on fewer points. Neither of these conditions is likely to occur, except when a watercourse is the obstacle to be surmounted, when, probably, a navigable channel of certain width has to be maintained, with sufficient headway to admit of the free passage of vessels. If in any case it should be desirable for the span to be greater than the minimum amount dictated by the compliance with these conditions, it could only be when the depth and rapidity of the current, or the treacherous nature of the bottom, rendered it desirable to reduce the cost of the piers and the risk of construction to a minimum. As steep gradients are now worked with ease and economy, it is not at all probable that any other case—such as that of carrying a line across a ravine—will ever occur in which it would be economical to introduce longer spans than 300 ft.; and it is only spans above that amount which we designate *long* spans in the present paper. Again, it is highly improbable that any rigid long-span bridge should be other than a railway bridge, because the great expense involved in its construction must be justified by necessity; in other words, by the probability of such large traffic as a railway alone could accommodate.

Even when we have thus limited the question to railway bridges crossing a watercourse, where a given span and height have to be maintained, we have by no means obtained all the conditions enabling us to pronounce upon the proper type of construction to be adopted. Thus, if the banks of a river are lofty, and afford a firm foundation for the superstructure without the intervention of masonry piers, an arch or suspension bridge may possibly be the most economical construction, although the resulting span may be greater than absolutely required; and if the banks are nearly level with the stream, it may or it may not be advisable to make the adjacent spans of greater length and weight than would ordinarily be required, in order to enable them to contribute more effectively to the support of the larger centre span. In short, it is plain that the determination of the most economical construction for a bridge of given span is a problem admitting of no general solution. We may, however, facilitate the process much, and obtain valuable positive results, if we confine our attention at first to the comparative weights of iron required in different methods of constructing the superstructure, which, after all, is by far the most important element in the cost of a long-span bridge.

The size of a bridge is too commonly the popular standard by which the eminence of its engineer is measured. We may, therefore, naturally expect to find engineers ambitious of excelling one another in this respect, but, for the same reason, as so much consideration must necessarily at one time and another have been devoted to the elucidation of long-span bridges, a student of engineering may justly be surprised to find so little definite information existing as to the respective capabilities of different types of construction to do the required work effectively and economically. Yet the number of existing patents referring to this matter indicates clearly the want of appreciation of the fact that the problem is one admitting of a rigid theoretical solution, and that the limit beyond which the quantity of metal required in the actual construction exceeds the amount theoretically required will be a factor of the latter quantity, the value of which may be approximated to very nearly, if we avail ourselves of the stock of information afforded by existing though smaller structures.

Although, as we urge, it is possible to approximate very nearly to a true result in every case, a considerable amount of intricate calculation and considerable space would be required to treat the subject exhaustively. Yet a great deal may be done, with little demand upon the application of the reader, if we base our investigations on the simplest and broadest principles, avoiding all complications and neglecting altogether minute details. If we proceed thus, we may, by making, so to speak, a cartoon of the different systems, exhibit in bold outline the respective advantages and disadvantages appertaining to each. This will be our aim in the present paper. We shall investigate on the above broad principles the weights of different types of girders, including all probable combinations, from the minimum span of 300 feet to the limiting span, beyond which it would be impossible to construct a bridge of the class capable of carrying more than its own weight without exceeding the given limiting strain per square inch. We shall carry out similar investigations both for iron and steel, and so conduct them, that by arranging the results graphically in the form of a diagram, we may obtain a comprehensive view of the properties of the different designs, and the nature of the laws governing the increase of weight, and consequent relative cost of the different constructions in the two materials.

The general principles on which we shall proceed are identical with those already advanced by the author in a paper on "The Proper Depth of Girders."* We shall, where we consider it advisable, sacrifice mathematical exactness of formulæ to simplicity, and generally allow a very free interpretation of theoretical deductions; the numerical results will be worked out with the slide rule, and, in short, the process throughout will be consistent with our professed object of exhibiting a cartoon-like view of the subject under consideration.

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* 'On the Strength of Beams, Columns, and Arches.' London: Spon.

In the paper alluded to, we observed that the maximum strain on each flange of a girder of uniform depth is, by the simple principles of leverage, equal to the distributed load multiplied by the span, and divided by eight times the depth, the strain being greatest at the centre, and less elsewhere in proportion to the ordinates of a parabola; and as this strain on the flange could only have been transmitted through the web, a little consideration will make it evident that for a distributed load, whatever the resultant strain may be, the total amount on the half web, resolved horizontally, will be equal to the maximum strain on the flange, or, in other words, the horizontal sectional area of a plate web must be at least equal to double the sectional area of the flange at the centre. For a lattice web the joint sectional area of the diagonals must be greater than this in the same ratio as the length of one of the bars exceeds the horizontal distance included between its two ends, which for the most economical angle of 45° for the bars amounts to $\sqrt{2}$ times the section required in a plate web. Hence, as both the length and area of the bars vary as the $\sqrt{2}$, the mass will be $\sqrt{2} \times \sqrt{2}$, or double that of the plate web. Expressed algebraically, S being the span, and *d* the depth in feet, *a* the sectional area of *each* flange, and *x* and *y* coefficients depending upon the practical construction of the flange and web respectively, the mass of the plate girder will be $= 2a(Sx + yd)$, and that of the lattice girder will be $= 2a(Sx + 2yd)$.

With the plate web, we found the varying economic ratio of $d$ to S to be governed, chiefly, by the minimum thickness of plate allowed in the construction of the web, which we considered should increase with additional loads per foot on the girder. With the lattice girder, we considered that, whilst the weight of the flange would in all cases be inversely proportional to the depth, that of the web, beyond certain limits, would increase directly as the depth; that is, although theoretically constant, there would be some limit beyond which the additional strength required in the struts as columns would necessitate a greater quantity of metal than the shorter struts with the heavier direct compression. This limit we have in all instances considered to be reached when the weight of the web becomes equal to that of the flanges. Hence, since the mass of the flanges, $2\,S\,x\,a$, varies inversely, and that of the web, $4\,y\,d\,a$, directly as the depth, the minimum value of the whole mass will occur when $2Sx = 4yd$. The effect of this in the different constructions will be controlled and adjusted for each case by the variable values of the coefficients $x$ and $y$.

In order to arrive at the weight of iron required in the construction of a girder capable of carrying a given load with a given maximum strain per square inch, we shall find it convenient to invert somewhat the ordinary mode of procedure. Thus, instead of starting with the load and determining the strain from that datum, we shall deduce the former element from the latter. Taking 1 square inch area of flange as the section resisting the maximum horizontal strain on that member, the mass of the lattice girder will be $2(\mathrm{S}x + 2yd)$; and from that we can easily determine the moment and the strain on the flange due to the load of the girder itself, which would obviously be the same amount per unit of area, whatever the gross section of the flange might be. We shall then have as data the limiting strain per square inch due to the entire load $= \mathrm{T}$, the strain per square inch due to the weight of the girder itself $= t$, and, consequently, we shall also have $\mathrm{T} - t$, the strain available for the useful load carried. Again, it follows that—moment of weight of girder : moment of useful load : : $t : \mathrm{T} - t$; hence, if the weight of the girder be uniformly distributed in the same manner as the remaining load, we have—weight of girder : useful load : : $t : \mathrm{T} - t$. Or, in other words, the weight of iron required in the construction of a girder to carry a given load will be the multiple $\frac{t}{\mathrm{T} - t}$ of that load.

By adopting the above method, we have much facilitated the solution of the problems before us, as we can now proceed with our investigations without complicating the question by the introduction of the varying loads, to which the bridges in each individual case will be liable.

The types of construction to which either alone or collectively we consider all forms of rigid bridges, not absolute eccentricities, may be referred, and to which, consequently, we have confined our investigations, are the following:

1. Box-plate girders, including tubular bridges.
2. Lattice do. do. Warren truss, &c.
3. Bowstring do. do. Saltash type.
4. Straight links and boom. Bollman truss.
5. Cantilever lattice uniform depth.
6. Do. do. varying economic depth.
7. Continuous do. do. do.
8. Arched ribs with braced spandrils.
9. Suspension with lattice stiffening girders.
10. Suspended girders.
11. Straight link suspension.

The comparative weights of the above constructions, both in iron and steel, will be investigated; but we shall first complete the necessary calculations for obtaining the weight of each of them in iron before we deal with the more untried material, steel.

Commencing with the most unfavourable type for long-span railway bridges which it will be necessary for us to investigate—the box girder with plate webs—we might, without any preliminary calculation, and with a very little amount of consideration, foretell the uneconomical results which must necessarily follow the distribution of metal in such an unsuitable form. Thus, the economical depth will be much less, and consequently the sectional area of flange required for a given load will be much greater than in any other type of girder. Again, the amount of metal required to prevent the buckling of the deep thin plates would be nearly sufficient to form the struts of a lattice girder; therefore, the effective duty of such a web will be little more than the resistance it offers to tensile strains. But these strains may be more economically provided for by lattice bars than by a solid plate; for, in the first case, the section may always be made proportional to the strain on the individual bar, whilst in the latter instance a certain minimum thickness of plate must be maintained at all points, which involves a waste of metal throughout nearly the entire length of the girder. Now, the mass of a plate girder for each square inch sectional area of flange at centre we have found to be equal to 2 $(S x + y d)$. Taking the weight of a bar of iron 1 ft. long and 1 in. square at ·03 cwt., the above mass multiplied by ·03 will give the weight of the girder in cwts. for each square inch section of flange, and the weight multiplied by $\frac{1}{8}$ span will give the moment $\mu$.

$$\mu = \cdot 03 \frac{S^2 x \times S y d}{4}.$$

Again, the strain, $t$, in cwts. per square inch resulting from the weight of the girder itself will be $\frac{\mu}{d}$; hence, since $S x = S y d$ when $d$ is the most economical depth, we have:

$$t = \frac{\cdot 015\, S^2 x}{d}.$$

Now, the influence of the weight of the web is the most important element in determining the proper depth for a plate girder, because whilst all the disturbing influences affecting the flanges also affect the web, there is, in addition, another element introduced, namely, the limiting thickness below which the plates may not be reduced. This is never taken at less than $\frac{1}{4}$ in., and in situations not easily accessible for the purpose of painting this thickness should be increased to $\frac{3}{8}$ in., and when the load per foot is large, a thicker plate is usually employed. But the effective horizontal section in square inches of a web of uniform thickness, taking, as in the case of the flanges, a reduced strain to compensate for the loss of section through the rivet-holes, will, since the strain increases uniformly from the centre to the ends, and the span, S, is in feet, be equal to 12 S $\times \frac{1}{2} \times \frac{1}{2}$ thickness $= 3$ S $\times$ thickness; consequently, as the least thickness of plate is $\frac{1}{4}$ in., the least effective horizontal section of any web will be $\frac{3}{4}$ S; and there can be but a small reduction in the weight of a web of uniform thickness, whatever may be the lightness of the load.

As far as the web is concerned, there would obviously be a practical advantage in making the depth of a girder small in proportion to the span, but as the small depth is a disadvantage to the flanges, the determination of the depth at which the joint weights of the flanges and web will be a minimum, is the problem to be solved.

Now, W being the distributed load, the other notation remaining as before, the sectional area ($a$) at the centre of the flange, in square inches, will be $a = \frac{\mathrm{W\,S}}{8\,d\,\mathrm{T}}$, which amount will also represent the actual horizontal section required in the half web; but we have seen that in practice this latter area is never less than $\frac{3}{4}$ S; consequently, the value of $a$ for the web must never be taken at *less* than that amount.

As we know the mass, and consequently the weight, of a girder to be proportioned to $a\,(\mathrm{S}\,x + y\,d)$, it is only necessary now to ascertain to what extent the limiting value of $a$ for the web will affect the question.

Now, taking a web of uniform thickness, and adopting the highest value of $y$ and the lowest of $x$, it is obvious that, if the mass of the flanges exceeds that of the web, the depth must necessarily be too small, since an increased depth would similarly affect the weight of the web directly, and the flange inversely; thus, assuming the mass of the flange to be that of the web as 6 : 4, the sum being 10, then, if the depth were increased $\frac{1}{4}$, the mass of the web would be $\frac{4 \times 5}{4} = 5$; and that of the flange $\frac{6 \times 4}{5} = 4 \cdot 8$, the sum being $9 \cdot 8$. Again, taking the highest value of $x$ and the lowest of $y$, we can arrive in the same manner at the maximum depth. If the web be not of uniform thickness, it is even more apparent that an excess in the mass of the flanges over that of the web indicates deficient depth, since increased depth would involve a proportionally less increase in the weight of the web. We have, therefore, $a\,x\,\mathrm{S} = a_1\,y\,d$, and we know that $a_1$ can never be less than $\frac{3}{4}$ S, and never be more than $\frac{\mathrm{W\,S}}{8\,d\,\mathrm{T}}$. Now, supposing that the thickness of the web might be reduced indefinitely, then, in order for the mass of the flange to be equal to that of the web, the span must obviously be to the depth as $y : x$; thus, theoretically, the flange and web will be of similar weights when the depth equals $\frac{2}{3}$ of the span. Although we cannot reduce the thickness of the web below a certain amount, the variations in the value of $y$ are too small to affect materially the economy of using as thin a web as possible. Taking a web $\frac{1}{4}$ in. thick, therefore, we have $a\,x\,\mathrm{S} = \frac{3}{4}\,\mathrm{S}\,y\,d$, and

$a = \frac{W\,S}{8\,d\,T}$; therefore, $W\,S\,x = 6\,d^2\,T\,y$ and $d^2 = \frac{W\,S\,x}{6\,T\,y}$.

We have previously observed that a thicker plate than $\frac{1}{4}$ in. is desirable if the load per foot exceeds a certain amount, the limits of which must of necessity be determined somewhat arbitrarily; this will, however, only affect the value of the constant (6).

Taking $\frac{1}{4}$ in. bare as the minimum thickness of plate allowable in the construction of a box girder 300 ft. span, and $\frac{1}{2}$ in. full as that necessary for a similar girder 600 ft. span, the value of the above constant ($c$) may be taken as 10 for the 300 ft. span, and 26 for the 600 ft. span. Generally, we may put $c = 3 \cdot 5 + \frac{W}{10\,S}$. From existing structures we deduce the values of $x$ and $y$ to be respectively $x = \cdot 93$, and $y = 5 \cdot 4$ for short spans, increasing in a certain ratio with the span, on account of the extra amount of bracing required; say, $y = 5 \cdot 4 + \cdot 002\,S$.

With a useful load of 35 cwt. per foot to be carried by each girder, which will be a sufficient approximation to the truth for our present purpose, the gross load upon the girder will be:

$$W = \frac{35\,T\,S}{T - t}.$$

We have, therefore, the strain in cwts. per square on the flange of a box girder due to its own weight:

$$t = \frac{\cdot 015\,S^2\,x}{d},$$

when $x = \cdot 93$, and $y = 5 \cdot 4 + \cdot 002\,S$.

$$d = \sqrt{\frac{W\,S\,x}{T\,c\,y}}; \quad W = \frac{35\,T\,S}{T - t}, \text{ and } c = 3 \cdot 5 + \frac{W}{10\,S}.$$

Adopting a working strain of 80 cwt. per square inch, and substituting and reducing, we obtain:

$$t = \sqrt{162\,n + \frac{n^2}{4}} - \frac{n}{2}.$$

when $n = \dfrac{2640\,S^2 + S^3}{23{,}700{,}000}$.

Applying this formula to the given spans, we arrive at the results shown in the following Table:

| Span in feet. | Strain in cwts. per sq. in. | Economic depth. |
|---|---|---|
| 300 | 37 | $\frac{1}{9}$ span |
| 400 | 48 | |
| 500 | 58 | |
| 600 | 68 | |
| 700 | 77 | |
| 800 | 85 | $\frac{1}{8}$ span |

As we have now before us the strains per square inch on the girders resulting from their own weights, we can, by the methods already shown, at once ascertain the weight of iron required to carry a given load, since it will be expressed in terms of that load by the multiple $\frac{t}{T-t}$, the value of which for each span enumerated is shown below:

| Span in feet. | | | | Multiple. |
|---|---|---|---|---|
| 300 | × | $\frac{37}{43}$ | = | ·86 |
| 400 | × | $\frac{46}{32}$ | = | 1·5 |
| 500 | × | $\frac{58}{22}$ | = | 2·6 |
| 600 | × | $\frac{68}{12}$ | = | 5·6 |
| 700 | × | $\frac{77}{3}$ | = | 25·6 |
| 800 | | | | ∞ |

We shall defer for the present any consideration of the probable loads to which the railway bridges of the above spans will be subject, and, necessarily also, of the actual weight of iron required in each instance. It will be found more convenient to treat our type girders collectively, with reference to the load; we shall, therefore, first advance them all as far as the preceding stage.

In the course of our present investigations, we shall find repeated instances of theoretical advantage in form being overruled, and more than neutralized, by some practical disadvantage incidental to the construction of the girder. The type we have already considered is a case in point. Theoretically the plate web girder requires less metal than any other form, and next to it in economy ranks the lattice girder with bars at the angle of 45°. Now, in practice we find these conditions to be precisely reversed, the plate web ranking lowest, whilst, as we shall hereafter show, the lattice girder is superior to that type alone in the scale of economy.

The practical advantage of the lattice girder as compared with the plate girder is due to the greater depth which may economically be employed in the former system, and not to the smaller quantity of metal required in girders of equal depth in both instances. The want of a correct appreciation of this fact, or the way in which it is commonly ignored, is evidenced but too forcibly in the massive stunted lattice girders so prevalent on English railways; and it is no just cause for surprise that girders of such proportions should not compete successfully on the score of economy with the spider-like trussing of American lattice bridges.

Although the stiffness of a lattice girder is less than that of a box girder of similar depth, the stiffness of the type lattice girder will be greater than that of the type box girder, on account of the greater depth obtained in the former construction.

Adopting a similar mode of procedure to that already exhibited in the instance of the box girder, we have for

### Type 2.—*Lattice Girder.*

Mass of girder for each square inch section of flange at centre proportional to $2\,(\mathrm{S}\,x + 2\,y\,d)$; which, multiplied by $\frac{1}{8}$ span and by $\cdot 03$ cwt., will give the moment:

$$\mu = \cdot 03\,\frac{\mathrm{S}^2 x + 2\,\mathrm{S}\,y\,d}{4}.$$

But $\frac{\mu}{d}$ = strain per square inch, and since $\mathrm{S}\,x = 2\,y\,d$, the economic depth will be $d = \frac{\mathrm{S}\,x}{2\,y}$. By substitution we have, therefore, the strain in cwts. per square inch, due to the weight of the girder itself:

$$t = \cdot 03\,\mathrm{S}\,y.$$

Availing ourselves, as before, of the data afforded by existing girders, we have $x = \cdot 93$, and $y = 2 \cdot 7 + \cdot 001\,\mathrm{S}$; hence

$$t = \cdot 081\,\mathrm{S} + \cdot 00003\,\mathrm{S}^2.$$

It appears also that the economic depth diminishes from $\frac{1}{6\cdot 5}$ of the span at 300 feet, to $\frac{1}{7\cdot 5}$ of the span at 800, thus showing the operation of a different law to that exhibited in the instance of the box girder.

The following Table shows the strain in cwts. per square inch, resulting from the weight of the girder itself in each instance:

| Span. | Strain in cwts. per sq. in. | Economic depth. |
|---|---|---|
| 300 | 27 | $\frac{1}{8\cdot5}$ |
| 400 | 37·2 | |
| 500 | 48 | $d=\frac{Sx}{2y}$. |
| 600 | 59·4 | |
| 700 | 71·4 | |
| 800 | 84 | $\frac{1}{7\cdot5}$ span. |

Since the weight of the girder itself and of its load is similarly distributed, the amount of the former weight will be the multiple $\frac{t}{T-t}$ of the latter. The results for the several spans are given below:

| Span. | | | Multiple. |
|---|---|---|---|
| 300 | $\frac{27}{53}$ | = | ·51 |
| 400 | $\frac{37\cdot2}{42\cdot8}$ | = | ·87 |
| 500 | $\frac{48}{32}$ | = | 1·5 |
| 600 | $\frac{59\cdot4}{20\cdot6}$ | = | 2·83 |
| 700 | $\frac{71\cdot4}{8\cdot6}$ | = | 8·3 |
| 800 | | = | ∞ |

## Type 3.—*Bowstring Girders.*

The bowstring girder consists of an arched rib to carry the load, and a tie, instead of the usual abutments, to resist the thrust of the arch, and prevent the tendency to spread at the feet. If the line passing through the centre of gravity of all the cross sections of the arched rib corresponds in position with the curve of equilibrium due to the distribution of the load, the only connection necessary between the arch and the tie will be such vertical ties as may be required to transmit the weight of the tie and its insistent load to the arched rib. But, as the condition of stability of the arched rib requires that the curve of equilibrium of the load, as transmitted to it, should correspond in form with the arch itself, it follows that, if the distribution of the load on the platform of the bridge be different to that required, the mere insertion of vertical ties capable of transmitting the load in direct lines only will not be sufficient. It will obviously be necessary to supply diagonal members capable of transmitting the load to the points in the arch where it is required to effect compliance with the conditions of equilibrium.

It appears, therefore, that, as the ordinary segmental arched rib of a bowstring bridge corresponds in form very nearly with the parabolic curve of equilibrium due to a distributed load, with such a load no diagonal ties are necessary. It does not necessarily follow that if ties were inserted the strain on them would be *nil;* in point of fact, the deflection of the girder would communicate a certain amount of strain to those members. If, however, the load be rolling, it will be absolutely necessary to supply such an amount of bracing as may be required to effect the equal distribution of the load on the arched rib, whatever may be its actual position on the platform of the bridge.

In either case, however, the quantity of metal introduced will be small compared with the amount absorbed in the corresponding "web" portion of a parallel girder. We may, therefore, make a bowstring girder of greater depth than could economically be adopted in the box or lattice construction. That is the reason why in every instance a bowstring girder may be constructed of no greater weight than a plate or lattice girder of corresponding strength; and why, in cases where the dead load is great, or, in other words, when the span is long, it may even be constructed with a much smaller amount of metal, notwithstanding the heavy theoretical disadvantage the bowstring girder labours under in the diminished depth towards the ends, and the consequent increase, instead of diminution, of sectional area of the arched rib at those points.

The stiffness of a bowstring girder will be less than that of a lattice girder of similar depth and strength. If there were no diagonal members in the construction, the deflection of the arched rib alone, supposing the abutments fixed, would be nearly equal to that of the lattice girder; whilst, as the abutments are movable to the extent of the extension of the tie, that amount of deflection would be about doubled. The introduction of bracing would diminish this deflection in proportion to the strength and adjustment of its several parts. With the type bowstring girder this condition taken in connection with the increased depth will diminish the deflection to about the same amount as that obtained in the instance of the type lattice girder.

From what we have already observed, it may easily be seen that the weight of iron required in a bowstring bridge to sustain a given load will be governed chiefly by the amount, but partly also by the nature, of the load. In a railway bridge the load is of a mixed character, consisting of dead and rolling elements in varying proportions. We shall therefore first deal with the two extreme conditions—all dead and all rolling loads—and ascertain the comparative quantity of metal required in each. To obtain the quantity necessary to carry a mixed load, we shall merely combine the preceding amounts in the same proportions as the dead and rolling elements obtain in the total load. This, of course, is not a correct method theoretically, as we are adding together + and − strains on some of the members. We have, however, in this instance, as in several others, in accordance with our professed intention of avoiding all complications, preferred keeping the question in its simplest form; and as the required corrections are effected by means of the coefficient $y$, the final result obtained will be correct, although the process by which it was arrived at is not quite so.

### *Dead Load* (*uniformly distributed*).

The mass of the tie will be proportional to $a$ S simply, and the mass of the arched rib will be equal to that of the tie, *plus* a certain additional amount due to the shearing strain transmitted through the arch. The shearing force in action consists of the entire weight of the load, *minus* a certain proportion of it which may be resting immediately on the piers without being first transmitted through the arched rib. Now, instead of ascertaining the increased sectional area required towards the ends of the arched rib in consequence of the combined action of the uniform horizontal force and of the uniformly increasing shearing force, and multiplying the mean sectional area thus determined by the length of the curved rib for its mass, we may approximate to the same result very nearly, and by a very simple process, if we keep the two forces in action distinct throughout, and deal with their masses separately. Thus we shall imagine the arched rib to be replaced by a horizontal member to resist the horizontal force, and by a vertical member equal in height to the depth of the girder to resist the shearing force.

The mass of the horizontal member will, of course, be the same as that of the tie $= a\,\mathrm{S}$, and the mass of the vertical member will, if we take $\frac{7}{8}$ of the gross load as the amount transmitted through the arch, be to that of the tie as $\frac{7\,d^2}{\mathrm{S}} : \mathrm{S}$. The total mass of the arched rib and tie will therefore be equal to,

$$a\left(2\,\mathrm{S} + \frac{7\,d^2}{\mathrm{S}}\right).$$

Again, the mass of the vertical members transmitting the weight of the tie and its insistent load to the arched rib may, for our purpose, be taken as equal to $\frac{2\,d^2}{\mathrm{S}}$; consequently, if $x$ and $y$ be coefficients for the horizontal and vertical members respectively representing the ratio of the metal used in practice to that theoretically required, the total mass of the bowstring girder for a uniform load will be

$$a\left(2\,\mathrm{S}\,x + \frac{9\,d^2\,y}{\mathrm{S}}\right).$$

But the mass multiplied by $\frac{1}{8}$ span and by ·03 cwt., will give the moment in cwts. for each square inch sectional area of tie; hence for 1 square inch,

$$\mu = \cdot 03\left(\frac{\mathrm{S}^2\,x}{4} + \frac{9\,d^2\,y}{8}\right).$$

Again, since $\frac{\mu}{d}$ = strain in cwts. per square inch; and economic $d^2 = \frac{2\,\mathrm{S}^2\,x}{9\,y}$, we have $d = \frac{7}{15}\,\mathrm{S}\sqrt{\frac{x}{y}}$; and the strain in cwts. per square inch due to the weight of the girder itself,

$$t = \cdot 03 \frac{15}{14} \mathrm{S} x \sqrt{\frac{y}{x}}.$$

*Rolling Load.*

As the maximum strains on both the arched rib and the tie occur when the rolling load entirely covers the bridge, it is obvious that the mass of those members will be the same as for a distributed load

$$= a\left(2\mathrm{S} + \frac{7d^2}{\mathrm{S}}\right).$$

The *web* portion, however, as we have already shown, will have a different duty to perform; and it is necessary now to ascertain to what extent this will affect the mass of the girder.

Now, it may be shown that the maximum horizontal strain to which the diagonals of a bowstring bridge are liable is the same for each bar; and that, assuming 10 bays of diagonals, the strain in each instance will be about $\frac{1}{10}$ of the maximum horizontal strain on the arched rib or tie, due to the rolling load uniformly distributed. The mass, therefore, of the horizontal components of one complete set of 10 diagonals will be $a\frac{\mathrm{S}}{10}$.

Again, the mean square of the verticals being $\frac{d^2}{2}$ nearly, and there being 10 bays, the mass of the vertical components of the said diagonals will be $a\frac{5d^2}{\mathrm{S}}$ nearly; consequently, the total mass of one set of diagonals will be:

$$a\left(\frac{5\,d^2}{S}+\frac{S}{10}\right)\text{ nearly.}$$

As we have assumed two sets of diagonals, one crossing the other, and one set of struts, the total mass of the *web* portion of the girder will be,

$$a\left(\frac{14\,d^2}{S}+\frac{S}{5}\right).$$

Taking the coefficients $x$ and $y$ of the same value as before, we have the mass of the bowstring girder for a pure rolling load equal to,

$$a\left(\frac{11\,S\,x}{5}+\frac{21\,d^2\,y}{S}\right).$$

Now, the moment in cwts. equals the mass multiplied by $\frac{1}{8}$ span and by $\cdot 03$ cwt.; hence we have for 1 square inch area of tie:

$$\mu = \cdot 03\left(\frac{11\,S^2\,x}{40}+\frac{21\,d^2\,y}{S}\right).$$

But $\frac{\mu}{d}$ = strain in cwts. per square inch, and $d^2 = \frac{11\,S^2\,x}{105\,y}$; hence $d = \frac{10}{31}\,S\sqrt{\frac{x}{y}}$; and, by substitution, the strain due to the weight of the girder will be,

$$t = \cdot 03\,\frac{17}{10}\,S\,x\sqrt{\frac{y}{x}}.$$

## *Mixed Load.*

It appears, therefore, that the strain per square inch resulting from the weight of a bowstring girder constructed to carry a dead load only, will be to that occurring on an equally strong girder constructed for a rolling load of similar intensity as $\frac{15}{14} : \frac{17}{10}$. Now, excluding the weight of the girders, the load to be carried by a railway bridge may be taken as consisting of $\frac{3}{4}$ rolling and $\frac{1}{4}$ dead load; consequently the equivalent fraction for the mixed load will be $\frac{17}{10} \times \frac{3}{4} + \frac{15}{14} \times \frac{1}{4} = \frac{108}{70}$; or, taking the fraction $\frac{15}{14}$ for the dead load as the unit of measurement, that for the mixed load will be 1·44 times the amount. Since for a dead load,

$$t = \cdot 03\,\frac{15}{14}\,S\,x\sqrt{\frac{y}{x}},$$

we have when $x = 1\cdot 25$, and $y = 3\cdot 2 + \cdot 002\,S$:

$$t = \cdot 04\,S\sqrt{2\cdot 56 + \cdot 0016\,S};$$

and the strain in cwts. per square inch due to the weight of a bowstring girder to carry the mixed load on a railway bridge will be,

$$t_1 = \frac{1\cdot 44\,t\,T}{T + \cdot 44\,t}.$$

Taking the limiting strain, T = 80 cwt. per square inch, and arranging the results as before in a tabular form, we have:

| Span in feet. | | Strain in cwts. per sq. in. | Depth. |
|---|---|---|---|
| 300 | = | 27 | $\frac{1}{6}$ span |
| 400 | = | 35·5 | |
| 500 | = | 43·5 | |
| 600 | = | 52 | |
| 700 | = | 60 | |
| 800 | = | 67 | |
| 900 | = | 75 | |
| 1000 | = | 80 | $\frac{1}{8}$ span |

The multiple $\frac{t_1}{T - t_1}$ will have the following values:

| Span in feet. | | | Multiple. |
|---|---|---|---|
| 300 | $\frac{27}{53}$ | = | ·51 |
| 400 | $\frac{35\cdot 5}{44\cdot 5}$ | = | ·79 |
| 500 | $\frac{43\cdot 5}{36\cdot 5}$ | = | 1·19 |
| 600 | $\frac{52}{28}$ | = | 1·86 |

| Span in feet. | | | Multiple. |
|---|---|---|---|
| 700 | $\frac{60}{20}$ | = | 3· |
| 800 | $\frac{67}{13}$ | = | 5·15 |
| 900 | $\frac{75}{5}$ | = | 15· |
| 1000 | | = | ∞ |

## Type 4.—*Straight Link Girders.*

We are not aware that any example of the above mode of construction exists in this country. The nearest allied to it, perhaps, is Brunel's Chepstow Bridge, and even that structure may be more properly referred to our second type, as in reality it is little more in principle than a three-bay lattice bridge, the most noticeable feature, and the one redeeming the design from the charge of extravagance, being its great depth, amounting to about ¼ the span.

In America, on the other hand, the straight-link girder, under the name of the Bollman truss, appears to meet with general approval; and, as there are several large bridges of the class erected in that country, we may consider the principle to have been fairly tested as to its practical capabilities.

Theoretically, our present type is the heaviest form of girder we have yet considered, and in the discussion at the Institution of Civil Engineers following Mr. Zerah Colburn's paper on American iron bridges, it was on such grounds condemned as uneconomical. We have, however, advanced sufficiently with our investigations to be satisfied how fallacious conclusions must be which rest on so uncertain a basis as mere theoretical considerations. All our types as yet have occupied precisely reverse positions in the scale of economy to that indicated for them respectively by theory; we may not, therefore, be surprised if we find this—the lowest in the theoretical scale—positively heading its competitors in practice.

In principle the straight-link girder is more analogous to the bowstring than to any other system. In both instances we have one straight member of uniform section throughout—the tie and the boom—opposed by a member of greater length and of increased section towards the piers—the arched rib and the collection of ties. We found the weight of the bowstring girder to vary considerably with the character of its load, and it will be seen hereafter that the same element influences the weight of the straight-link girder, the conditions, however, being reversed. From the nature of the trussing in the present case, a rolling load will be more economically dealt with than will the load due to the weight of the girder itself, whilst it will be remembered the former load operated disadvantageously on the bowstring girder. It follows from this that if the short-span bowstring girder enjoys any advantage over our present type, it will maintain a still greater advantage in the long spans, whilst on the other hand, if the straight-link girder excels the former system for the short spans, it by no means follows that it will be able to compete with it for the long spans. Indeed, theoretical deductions show it to be otherwise; and as in this instance they are not overruled by practical considerations, the fact of the moment of the load transmitted to the bars being $1\frac{1}{3}$ times that taking effect upon an arched rib under similar conditions, must detract from the economy of the straight-link girder, as compared with the former system when uniform loads are in question.

With reference to the comparative stiffness of this description of girder, it was remarked in the discussion at the Institution of Civil Engineers previously alluded to, that diagonals of great inclination are free to deflect on a curve struck from one of the ends as a centre, and the other end as a radius; and that, as the curve so described would coincide in practice for a considerable distance with a straight line, there would be little or no resistance to deflection. Now, this conclusion is palpably false, as the deflection will vary inversely as the angle included between the given pair of ties, and will not depend upon the inclination of either of them to the horizon. Thus, if the two bars are at right angles to each other, it is immaterial as regards deflection whether both of the bars incline at an angle of 45° to the horizon, or whether one is vertical and the other at the *flattest* possible angle, in fact, horizontal, since with equal depths and unit strains the deflections will be similar.

As the most obtuse angle included between any two bars in a straight-link girder occurs at the centre of the span, the deflection will be greatest at that point, and will be less towards the ends, as in other structures; the amount, however, in this instance will be about double that occurring on a lattice girder of similar depth and strain per square inch. In this respect it resembles the bowstring girder, but the practical disturbing elements reducing this double amount in the case of the bowstring girder do not obtain in the present instance.

There is a peculiarity, however, connected with the deflection of this girder, which should not be passed over without notice, as it may be of grave practical importance. As each pair of ties acts independently, affecting the others indirectly only through the medium of the boom, it follows that when any pair of ties sustain their full proportion of rolling load, they will also incur a large moiety of their full deflection, which deflection will not be shared, to any considerable degree, by the remaining unloaded portions of the bridge. The practical effect of this condition is, that as the rolling load advances along the bridge, it will, so to speak, break the back of the boom at the vertical springing from the nearest adjacent unloaded pair of ties.

With the 300 ft. span bridge, for example, the load being half over, the deflection at the centre would be about 3 in., whilst, if the bridge consisted of 10 bays, at a point 30 ft. off, the deflection would be only 1 in. In the length of 30 ft., therefore, we have to dispose of a difference in level of 2 in., about; and this will necessitate either an elastic boom subject to transverse strain, or else one jointed at each vertical. If we attempt to get over the difficulty by the insertion of bracing to equalize the deflection, as is sometimes done in the American bridges, there will be little hope of attaining an economical structure.

Having pointed out the foregoing practical difficulty in the construction of the straight-link girder, we shall assume it to be surmounted, without producing any abnormal strain on the boom, and without any extra provision of bracing, and shall now proceed to ascertain the quantity of metal required on that hypothesis.

### *Dead Load.*

The mass of the boom will be proportional to $a$ S, and the mass of the ties will be equal to that of the boom, together with the additional amount due to the transmission of the shearing strains through those members.

We shall deal with the masses required to resist the horizontal and vertical strains separately, as we did in the instance of the bowstring girder.

Now, taking the cluster of ties collected at each pier, it may be shown that the horizontal strain on each will be proportional to the ordinates of a parabola, of which the height represents the horizontal strain on the centre pair of ties. The mean height of all the ordinates of a parabola being equal to ⅔ of the central height, it follows that the mean horizontal strain on each bar will be ⅔ of the amount on the centre bars. But the moment of the load on the centre pair of ties is equal to $W\frac{S}{4}$; consequently the mean moment of all the ties will be ⅔ of that amount $= W\frac{S}{6}$. It appears, therefore, that with similar depths, loads, and unit strains, the sectional area of the *flange* portions of a straight-link girder will be $\frac{4}{3}$ of that necessary in either of the types we have yet considered.

The mass of the horizontal components of the ties being $a\,S$, the mass of the vertical components will, if we take ⅞ of the total load as transmitted through them, be to that amount as $\frac{21\,d^2}{4\,S} : S$. The total mass of the ties and boom will therefore be $= a\left(2\,S + \frac{21\,d^2}{4\,S}\right)$.

But we must also provide vertical members to support the boom and ties at certain intervals, the mass of which members may be taken as equal to $\frac{6\,d^2}{S}$; therefore, taking $x$ and $y$ coefficients for the horizontal and vertical members respectively, the total mass of metal required in the construction of a straight-link girder for a dead load will be,

$$a\left(2\,S\,x + \frac{45\,d^2\,y}{4\,S}\right).$$

Taking $a = 1$ square inch, the mass multiplied by ·03 cwt. and by $\frac{1}{8}$ span will equal the moment in cwts.

$$\mu = \cdot 03\left(\frac{S^2\,x}{3} + \frac{15\,d^2\,y}{8}\right).$$

But $\frac{\mu}{d}$ = strain in cwts. per square inch; and since economic $d^2 = \frac{8\,S^2\,x}{45\,y}$, we have $d = \frac{S}{2\cdot 37}\sqrt{\frac{x}{y}}$; and the strain in cwts. per square inch, resulting from the weight of the girder itself, equal to,

$$t = \cdot 03\left(1\cdot 58\,S\,x\sqrt{\frac{y}{x}}\right).$$

### *Rolling Load.*

As the maximum strain on the ties and boom is attained when the bridge is entirely loaded, the mass of those members will be the same as before,

$$= a\left(2\,S + \frac{21\,d^2}{4\,S}\right);$$

and, as we have assumed the platform of the bridge to be at the level of the bottom of the girder, no additional metal will be required to complete the girder for the rolling load. Therefore, $x$ and $y$ being the coefficients as before, the total mass will be:

$$a\left(2\,S\,x + \frac{21\,d^2\,y}{4\,S}\right).$$

And, for 1 square inch area of boom, the moment in cwts. = mass × ·03 cwt. × $\frac{1}{8}$ span will be:

$$\mu = \cdot 03\left(\frac{S^2 x}{3} + \frac{7\, d^2 y}{8}\right).$$

But $\frac{\mu}{d}$ = strain in cwts. per square inch; and since economic $d^2 = \frac{8\, S^2 x}{21\, y}$, $d = \frac{S}{1 \cdot 62}\sqrt{\frac{x}{y}}$; and the strain in cwts. per square inch due to the weight of the girder itself will be:

$$t = \cdot 03\left(1 \cdot 08\, S\, x \sqrt{\frac{y}{x}}\right).$$

## *Mixed Load.*

Proceeding as in the instance of the bowstring girder, we have $t = \cdot 03\left(1 \cdot 58\, S\, x\sqrt{\frac{y}{x}}\right)$; hence, when $x = 1 \cdot 25$, and $y = 3 \cdot 2 + \cdot 002\, S$,

$$t = \cdot 059\, S \sqrt{2 \cdot 56 + \cdot 0016\, S};$$

and the strain in cwts. per square inch due to the weight of a straight-link girder for a railway bridge, will be:

$$t_1 = \frac{\cdot 7\, t\, T}{T - 3\, t}.$$

With the limiting strain, T = 80 cwt. per square inch, we obtain the following results:

| Span in feet. | Strain in cwts. per sq. in. | Depth. |
|---|---|---|
| 300 | 23·1 | $\frac{1}{4}$ |
| 400 | 33·5 | |
| 500 | 45·0 | |
| 600 | 57·7 | |
| 700 | 72·7 | |
| 800 | 92·5 | $\frac{1}{5}$ |

The multiple $\frac{t_1}{T - t_1}$ will have the following values:

| Span in feet. | | | Multiple. |
|---|---|---|---|
| 300 | $\frac{23 \cdot 1}{56 \cdot 9}$ | = | ·4 |
| 400 | $\frac{33 \cdot 5}{46 \cdot 5}$ | = | ·72 |
| 500 | $\frac{45}{35}$ | = | 1·28 |
| 600 | $\frac{57 \cdot 7}{22 \cdot 3}$ | = | 2·61 |
| 700 | $\frac{72 \cdot 7}{7 \cdot 3}$ | = | 9·97 |
| 800 | | = | ∞ |

We have now arrived at the conclusion of what may be considered the first stage of our investigations. Our type girders, Nos. 1, 2, 3, and 4, are all independent structures, carrying their loads without any extraneous assistance, the only requisite being a supporting pier at each end capable of bearing one-half the maximum load on the bridge. As, in our opinion, all justifiable modes of constructing independent girders may be referred to one or the other of the preceding types, and will be included within the limits thereby defined, we shall now proceed to the second stage of our inquiry, which refers to structures whose stability depends upon some support beyond that afforded by the simple pier. We shall first consider those structures which—although dependent upon external assistance—produce only a vertical pressure on the piers; and, secondly, the systems whose stability is governed by the power of the piers, or abutments, to resist horizontal, as well as vertical forces.

The first division will comprise Types 5, 6, and 7, or, two kinds of cantilever girders and the continuous girder; and the second division will include the remaining type structures.

### Type 5.—*Cantilever Girders of uniform depth.*

In appearance, the type we have now to investigate is identical with the independent lattice girder, and this identity is not merely apparent, but does, in fact, obtain to a great extent in the web; but a little consideration will show that the flanges are placed under entirely different conditions.

In the independent girder of uniform depth, subjected to a distributed load, the strain on the flanges will be greatest at the centre of the span, and less elsewhere, in proportion to the ordinates of a parabola; whilst in a similar cantilever girder, although the maximum strain will be the same in *amount* as before, it will occur at the piers, and will diminish towards the centre in proportion to the co-ordinates of the same parabola; at the middle of the span the strain, therefore, will vanish. It follows that, in the independent girder, the theoretical mass of flange will be, to the product of the maximum sectional area by the length, as the area of a parabola: the area of the enclosing rectangle, or, as $\frac{2}{3} : 1$; whilst in the cantilever the proportion will be as the area of the complement of a parabola to the same rectangle, or, as $\frac{1}{3} : 1$. The mass of metal required theoretically to form the flanges of a cantilever will therefore be only one-half of that necessary in a similar independent girder, the load in both instances being uniformly distributed.

This proportion, however, does not represent the whole of the advantage accruing to the former system, as the *moment* of the flanges will be diminished in a much higher ratio than the *weight.*

The centre of gravity of the semi-parabola, representing the mass of metal required in the flanges on each side of the centre of an independent girder, being $\frac{5}{16}$ of the span from the pier, the moment of the flange will be proportional to $\frac{2}{3} \times \frac{5}{16} = \frac{5}{24}$; whilst the centre of gravity of the complement of the semi-parabola being $\frac{1}{8}$ of the span from the pier, the moment of the flanges of a cantilever girder will be proportional to $\frac{1}{3} \times \frac{1}{8} = \frac{1}{24}$. The ratio of 5 to 1 indicated by the preceding theoretical considerations is so high that we may be sure, after allowing an ample margin for all possible contingencies, a balance will still remain in favour of our present type, which must tell with considerable effect in the economy of long-span bridges, where a large proportion of the load consists of the weight of the girder itself.

Of course, whether the extra metal required in the adjacent spans, according to this system, may equal or exceed the saving effected in the main span, is another question, which does not concern us at present.

The maximum deflection of the cantilever girder will obviously be partly governed by the adjacent spans. If the side spans be one-half the main span, the deflection at the centre of the latter will be from 1½ times to double that of the independent girder, of equal depth and unit strains. The unequal deflection of the two halves of the main span, due to the passage of a rolling load, presents no practical difficulty, as a very simple connection may be contrived, admitting free vertical deflection and longitudinal expansion, but at the same time resisting any tendency to lateral movement.

Now the mass of the girder for each square inch area of the maximum cross section of flange will be the same as for Type 2, that is:

$$2\,(S\,x + 2\,y\,d).$$

Since the mass multiplied by ·03 cwt. and by one-half the distance of the centre of gravity of the half girder from the pier $\left(\frac{S}{b}\right)$ will give the moments in cwts., we have:

$$\mu = {\cdot}03\,\frac{2\,S^2 + 4\,S\,y\,d}{b}.$$

But $\frac{\mu}{d}$ = strain in cwts. per square inch ($t$); and since economic depth $= \frac{S\,x}{2\,y}$, we have:

$$t = {\cdot}03\,\frac{8\,S\,y}{b}.$$

The value of *b* will vary according to the distribution of the load; for an uniform load $b = 8$, and we must now ascertain its values for the various distributions of load obtained in railway bridges of different spans.

Now, if the load to be carried be uniformly distributed, the value of *b* for the portion of the load consisting of the weight of the girder itself will be as follows:

$$\left.\begin{array}{l}\text{Web} \;\;\;\;\; = 12\\ \text{Flanges} = 16\end{array}\right\}\begin{array}{l}\text{mean theoretical value } b = 14.\\ \;\;\text{,,} \;\;\;\; \text{practical} \;\;\;\;\;\;\; \text{,,} \;\;\; b = 11.\end{array}$$

Again, if the load consists of the weight of the girder alone, we have:

$$\left.\begin{array}{l}\text{Web} \;\;\;\;\; = 16\\ \text{Flanges} = 20\end{array}\right\}\begin{array}{l}\text{mean theoretical value } b = 18.\\ \;\;\text{,,} \;\;\;\; \text{practical} \;\;\;\;\;\;\; \text{,,} \;\;\; b = 15.\end{array}$$

It follows, therefore—T being the limiting strain in cwts. per square inch—the value of *b*, corresponding to the required strain, *t*, will be:

$$b = \frac{15\,t + 11\,(T - t)}{T}.$$

Substituting this value in the former equation, and taking $T = 80$ cwt., $y = 2{\cdot}5 + {\cdot}001\,S$, and $x = {\cdot}6$, we obtain:

$$t = \sqrt{12\,S + {\cdot}005\,S^2 + 12100} - 110;$$

which equation gives the following results:

| Span in feet. | Strain in cwts. per sq. in. | Depth. |
|---|---|---|
| 300 | 17·2 | $\frac{1}{9}$ |
| 400 | 23·2 | |
| 500 | 29·2 | |
| 600 | 33·1 | |
| 700 | 41·7 | |
| 800 | 48·0 | |
| 900 | 54·4 | |
| 1000 | 60·5 | |
| 1100 | 67·0 | |
| 1200 | 73·8 | |
| 1300 | 80·0 | $\frac{1}{12}$ |

We have already observed that for a distributed load, such as the weight on the platform of a railway bridge, $b = 8$; and we have also found its value for the girder weight required to carry this load at the various spans. We can, therefore, at once obtain the weight of iron required in the construction of the main girders, as it will be the multiple of the load expressed by the equation:

$$\text{Multiple} = \frac{t\,b}{8\,(T - t)}.$$

| Span in feet. | Multiple. |
|---|---|
| 300 | ·39 |
| 400 | ·61 |
| 500 | ·88 |
| 600 | 1·23 |

| Span in feet. | Multiple. |
|---|---|
| 700 | 1·64 |
| 800 | 2·5 |
| 900 | 3·6 |
| 1000 | 5·43 |
| 1100 | 9·25 |
| 1200 | 21·6 |
| 1300 | ∞ |

### Type 6.—*Cantilever Lattice Girder, varying economic depth.*

The most superficial examination of the method and results of our investigations relating to the cantilever lattice girder of uniform depth could hardly fail to suggest a desirable modification in its outline. Thus if we lay off 300 ft. span and plot the economic depth at each end, and then, adding 50 ft. to each end, plot the economic depth for 400 ft. span, we shall, if we carry on the process up to the limiting span, and connect the various plotted heights by lines, obtain a curve the ordinates of which will represent the economic depth for the different sections of the girder, and consequently, if there be no new condition introduced, of the entire girder.

But the alteration of the top flange from a straight line parallel to the bottom flange, to a curved line inclined to the latter, *does* introduce a new element into the case, as a portion of the shearing strain will now pass through the top flange, and to that extent, of course, the *web* will be relieved of strain.

Now the results of our last investigations indicate that the depth of a girder, and, necessarily, to a great extent its weight also, is governed by the mass of the *web*. It follows, therefore, that the more we can reduce the strain on and, consequently, the mass of, that member, the greater will be the economic depth, and, within certain limits, the smaller the total mass of metal required in the construction of the girder. It is not difficult, then, to see that it is desirable to adopt every available economical means of reducing the strain on the *web*. In cantilever girders, we can, fortunately, effect this by the very simple process of giving the upper flange a downward curvature. The tendency of the tension member to pull straight will react on the long struts, and by the production of an initial tension reduce the mass of metal required for those members of the web; whilst, if we make the curved outline *include* the various economic depths, we shall arrive at a stiffer form of girder than before.

It would be foreign to our present purpose, and inconsistent with our avowed intention of viewing our subject in the broadest possible light, were we to endeavour to deduce the precise amount of curvature which would give the most favourable general results. It will be a sufficiently near approximation to the correct average proportions, if we assume the depth at the centre of the girder to be one-fourth that at the ends, and the curvature of the top flange to be the segment of a circle passing through those three points.

The form of bridge to which we have been thus, as it were, irresistibly driven is, we believe, almost identical in outline and in general proportions with the structure designed by Mr. Fowler to span the 600 ft. centre opening, and the two 300 ft. side openings, of his viaduct for carrying the proposed "South Wales and Great Western direct" railway across the Severn estuary.

It will be unnecessary to give a detailed investigation of this modified form of the cantilever lattice girder.

The horizontal components of the diagonals will be, proportionally, the same as before; the vertical components, however, will be less, as a proportion of the shearing strain is transmitted through the curved top flange. The struts, again, will be lighter, on account of their diminished length, and, for the same reason, the flanges will be heavier. The centre of gravity of the mass of the entire girder being about the same as in the previous instance, we have merely to substitute new values of $x$ and $y$ in the equation already deduced for the parallel cantilever girder. Taking $x = \cdot 7$, and $y = 2 + \cdot 001\,S$, the strain in cwts. per square inch due to the weight of the girder itself will be:

$$t = \sqrt{9\cdot 6\,S + \cdot 004\,S^2 + 12100} - 110,$$

which equation gives the following results:

| Span in feet. | Strain in cwts. per sq. in. | Depth. |
|---|---|---|
| 300 | 14 | $\frac{1}{7}$ |
| 400 | 19 | |
| 500 | 24 | |
| 600 | 29 | |
| 700 | 34·5 | |
| 800 | 40 | |
| 900 | 45·5 | |
| 1000 | 51 | |
| 1100 | 56·5 | |
| 1200 | 62 | |
| 1300 | 68 | |
| 1400 | 74 | |
| 1500 | 80 | $\frac{1}{9}$ |

The multiple, as in last case, will be given by the equation:

$$\text{Multiple} = \frac{t\,b}{8\,(T - t)}\cdot$$

| Span in feet. | Multiple. |
|---|---|
| 300 | ·3 |
| 400 | ·46 |
| 500 | ·65 |
| 600 | ·88 |
| 700 | 1·2 |
| 800 | 1·61 |
| 900 | 2·17 |
| 1000 | 2·96 |
| 1100 | 4·15 |
| 1200 | 6·1 |
| 1300 | 11·2 |
| 1400 | 22·6 |
| 1500 | ∞ |

### TYPE 7.—*Continuous Girder, varying economic depth (including Sedley's patent).*

The preceding investigation has informed us that with the given unit strain, and an unlimited supply of metal, it is possible to construct a cantilever bridge of 1500 ft. span; it follows, as a corollary, that it is also possible to construct a cantilever, or bracket, of 750 ft. projection, capable of carrying any required load at its extremity. For, assume that we support this load by a simple triangular frame, consisting of an inclined tie and a horizontal strut, then the weight of this frame will not induce a strain on either tie or strut, since the whole affair will be borne by the original cantilever as a portion of its uniform load; and, consequently, it will be possible to construct this triangular bracket, or what amounts to the same thing, the original cantilever, of sufficient strength to carry any required load at its extremity. This being so, it necessarily follows that it will be practicable to rest the two ends of either of our type forms of independent girders upon the extremities of a pair of these cantilevers, as securely as if they rested on their original piers.

It is evident at once that if we *do* thus insert an independent girder between the two halves of a cantilever bridge, the limiting span of the entire structure will be equal to the sum of the limiting spans of the two systems of which the bridge is composed; thus, if we take our last type and the bowstring bridge, the limiting span will be 1500 + 1000 = 2500 ft.

Now, what conclusion must we draw from this fact? It appears we may on this system, with a definite amount of metal, bridge an opening which could not be spanned by either of the other systems we have yet investigated with an *infinite* quantity; and the irresistible conclusion is that at the high spans a much smaller amount of iron will be required in the construction of a "continuous" bridge of this class than would be necessary in one constructed on either of the other systems. This theoretical deduction is fully corroborated by the indisputable economy obtained in the bridges on this principle erected under Sedley's patent. The only thing we have to determine, then, is the span at which this superiority will begin to manifest itself, and that, of course, will vary with the degree of economy obtained in practice in the other systems with which it is to be compared. In our present investigation—as we have in all instances supplied a very liberal amount of metal for the construction of the several types—it will be when the sum of the multiples for the cantilever bracket and bowstring, reduced to the equivalent value when measured over the whole span, is smaller in amount than the multiple of the cantilever bridge for the same span. The Tables we have already calculated for the two systems of which this type consists will enable us to ascertain when this condition is obtained almost by inspection.

There is no practical difficulty in the construction of this compound structure calling for special notice. The bridge may be made in one connected length, and merely jointed at the points of contrary flexure occurring at the junctions of the bowstring with the cantilever, expansion being provided for at the piers in the usual way, or the bowstring may be slung from the ends of the cantilevers, and the expansion allowed to take effect at those points of the span. In the latter case, precautions must be taken to ensure the maintenance of the strength of the horizontal bracing past those points, so as to check all tendency to lateral movement.

The deflection will, of course, be the sum of the deflections of the two smaller spans into which the bridge may be broken up. This will give a rather smaller proportional deflection than that appertaining to either system individually. Now let $a$ = the sum of the lengths of the two cantilevers, and let $b$ = the span of the centre portion, or the bowstring girder; then $a + b$ = span of the "continuous" girder. Also let $n$ be the multiple corresponding to span $a$, and $m$ the multiple for span $b$, given in the tables of multiples for Types 6 and 3 respectively. Then the load per foot on the cantilever in terms of the *useful* load as before, reduced to the equivalent load per foot distributed over the entire span, will be $\frac{a(n+1)}{a+b}$. In the same manner the load from the bowstring will be $\frac{b(m+1)}{a+b}$; and assuming the weight on the end of the bracket as equivalent to double the amount distributed, the load from the "triangle" will be $\frac{2b(m+1)}{a+b}$. The sum of these amounts will be the mean equivalent load; and that amount, less the unit *useful* load, will be the mean equivalent multiple; therefore,

$$\text{Multiple} = \frac{a(n+1) + 3b(m+1)}{a+b}.$$

Substituting the values of $m$ and $n$ obtained from Tables 3 and 6, we find, by the preceding equation, that in higher spans than 550 ft. the "continuous" girder is lighter than the cantilever. At 550 ft. span, then, the economic span of the centre portion or the bowstring girder $= 0$; and we know that at 2500 ft. the economic span is also the limiting span $= 1000$ ft.

Now $\frac{2500-550}{1000} = 1 \cdot 95$; and it will be sufficiently accurate for our purpose if we assume the economic span of the centre portion generally to be:

$$\text{Economic span} = \frac{\text{Span} - 550}{2}.$$

Substituting this value of $b$ in our former equation, we obtain the following results:

| Span. | Multiple. | Span. | Multiple. |
|---|---|---|---|
| 300 | ·3 | 800 | 1·33 |
| 400 | ·46 | 900 | 1·61 |
| 500 | ·65 | 1000 | 1·84 |
| 600 | ·85 | 1100 | 2·1 |
| 700 | 1·1 | 1200 | 2·49 |

| Span. | Multiple. | Span. | Multiple. |
|---|---|---|---|
| 1300 | 2·87 | 2000 | 10·42 |
| 1400 | 3·37 | 2100 | 14·32 |
| 1500 | 3·91 | 2200 | 21·14 |
| 1600 | 4·58 | 2300 | 29·12 |
| 1700 | 5·35 | 2400 | 50 |
| 1800 | 6·47 | 2500 | ∞ |
| 1900 | 7·98 | | |

### Type 8.—*Arched Ribs with Braced Spandrils.*

With very few exceptions, all the earlier examples of what would, at the time of their erection, be considered long-span bridges were constructed on the principle of the arch. Indeed, in comparison with that system, all other types of construction are still in their infancy; and yet, perhaps, with reference to no other system does there exist so much indefiniteness and difference of opinion as to the real direction and amount of the strain occurring at any given point. The explanation of this is simple enough, since, before we can attempt to determine the strains on an ordinary arch, we must make certain assumptions with respect to the elasticity of the superstructure and abutments, the probable truth of which may be sufficiently proved in *our* opinion, but not so in the judgment of others. As we cannot absolutely demonstrate the truth of our hypothesis, anyone else is, of course, at perfect liberty to make a different one, which may, very probably, lead to essentially different results.

If, however, we have as *data*, the relative positions of any three points through which the centre of pressure on the arch passes, the locus of the curve is defined at every other point, just as the radius of a curve is defined when it has to intersect three given points. It follows that if we arrange the details of an arched rib so as to ensure the centre of pressure passing through three known points, we shall be enabled to determine all the conditions with the same precision, and to proportion the strength of the several members to the maximum strains occurring on each with the same accuracy as in the most elementary form of truss.

We have already adopted a similar method in the instance of the continuous girder, by first determining the most economical position for the points of contrary flexure, and then, by constructional arrangements, securing the constant position of those points under all conditions of loading. By this means we not only obtained a great theoretical advantage, but, by reducing the complicated and, in fact, almost indeterminate problem of the determination of the actual strains occurring on a continuous girder to the simple case of the independent girder, we were enabled in practice to effect an additional saving by proportioning the several members with much greater nicety to the maximum strains on each.

The desideratum in the design of an arched rib is, therefore, that the centre of pressure should, under all conditions of loading, pass through the same three given points. The most obvious way of effecting this is by making the arched rib movable on pivots at the centre and at each springing, thus hinging, as it were, the rib at those three points. If the frictional resistance to turning on these pivots be small—that is, if their diameters be small—the centre of pressure will always pass through their centres, which, for a symmetrical cross section of rib and uniform unit strain, should correspond with the axis of the rib.

Again, in this design, expansion and contraction will merely produce a rise or fall of the arch at the crown, without any incidental strain; whereas if the rib were continuous there would have been additional strain with the consequent loss of metal on that score.

We shall therefore confine our investigations to the arched rib jointed at the centre and at each springing. This, of course, assumes intermediate piers of sufficient stability to take up the unbalanced thrust due to the rolling load, if the arch be one of a series in a viaduct. If the piers be too lofty to admit of this necessary provision, we should adopt an arched rib of an entirely different pattern, which will be referred to in considering Type 10.

The problem reduced to its present dimensions is a very simple one, admitting of a definite solution; and on that account, partly, there is no reason why the æsthetically perfect form of the arch should not be economically employed in wrought-iron bridges.

The deflection of an arched rib will be very nearly the same as that of a lattice girder of similar depth and strain per square inch.

In considering the strains to which an arched rib is liable, it will be necessary to resolve the gross load into its two elements—dead and rolling; for in this instance, as in several previous ones, the strain will be to a great extent governed by the *nature* of the load as well as by its amount.

## *Dead Load.*

With a dead load uniformly distributed, it is not necessary for the spandril fillings of the arch to possess any bracing power, for the same reasons that the diagonals of the bowstring girder were dispensed with under similar conditions. The mass of the arched rib also will be the same as in the bowstring girder:

$$= a\left(S + \frac{7\,d^2}{S}\right);$$

and that of the verticals will be less, on account of their decreased length, the mass being about $\frac{a\,d}{S}$. Introducing the coefficients, $x$ and $y$, for the horizontal and vertical components of the strain, as before, the total mass of the arch for each square inch sectional area at crown will be:

$$S\,x + \frac{8\,d^2\,y}{S}.$$

But the mass, multiplied by ·03 cwt. and by $\frac{1}{8}$ span, equals the moment:

$$\mu = \cdot 03\left(\frac{S^2\,x}{8} + d^2\,y\right).$$

Since economic $d^2 = \frac{S^2\,x}{8\,y}$, we have depth $= \frac{S}{2\cdot 83}\sqrt{\frac{x}{y}}$; and since the strain in cwts. per square inch ($t$) equals $\frac{\mu}{d}$, we have for the dead load:

$$t = \cdot 03\left(\cdot 71\,S\,x\sqrt{\frac{y}{x}}\right).$$

## *Rolling Load.*

With a rolling load it will be necessary to introduce bracing between the arched rib and the "horizontal girder," of such strength that the moment of resistance to a transverse strain at any point of the spandrils may be equal to the bending moment at the same point, due to the unequal distribution of the load. We shall assume the depth of bracing at the centre of the span to be $\frac{1}{6}$ the rise of the arch, which will give us an effective depth of $\frac{1}{3}$ the rise at the point of maximum bending stress. This proportion will limit in all our examples the maximum strain on the arched rib to the same amount as it would occur were the load entirely dead. The mass, therefore, will be the same as before:

$$= a\left(S + \frac{7\,d^2}{S}\right).$$

Now the maximum bending moment will occur when the rolling load is half-way over the bridge, at which time it will, in terms of the bending moment on the entire span, be equal to $(\frac{1}{2})^3$. Since, however, the effective depth is only equal to $\frac{1}{3}$ the rise of the arch, the mass of metal required in the horizontal girder, or top member of the braced spandrils, will be equal to $(\frac{1}{2})^3 \div \frac{1}{3} = \frac{3}{8}\,a\,S$.

The mass required for the sum of the horizontal components of the strain on all the diagonals will, if we provide a double set of 10 bays of bars, be equal to $\frac{9\,a\,S}{20}$. Again, the mean square of the verticals being

$\frac{d^2}{2\cdot 6}$ about, it follows that the mass required in the vertical components of the same double set of diagonals will be $\frac{17\,d^2\,a}{S}$. The total mass of metal required in the arch for each square inch of sectional area at the crown will, therefore, be equal to:

$$1\cdot 8\,S\,x + \frac{24\,d^2\,y}{S}.$$

Since the mass multiplied by ·03 cwt. and by $\frac{1}{8}$ span equals the moment, we have:

$$\mu = \cdot 03\,(\cdot 225\,S^2\,x + 3\,d^2\,y).$$

But $\frac{\mu}{d}$ = strain in cwts. per square inch ($t$); and economic $d^2 = \frac{1\cdot 8\,S^2\,x}{24\,y}$; hence $d = \frac{3\,S}{18}\sqrt{\frac{x}{y}}$, and by substitution

$$t = \cdot 03\left(1\cdot 62\,S\,x\sqrt{\frac{y}{x}}\right).$$

### *Mixed Loads.*

Assuming, as before, the useful load on a railway bridge to be composed of $\frac{3}{4}$ rolling and $\frac{1}{4}$ dead load, the mean coefficient will be $1\cdot 62 \times \frac{3}{4} + \cdot 71 \times \frac{1}{4} \doteq 1\cdot 4$; or, taking the coefficient for the dead load $= \cdot 71$ as the unit of measurement, for the mixed load it will be about double that amount. Hence, since the strain in cwts. per square inch ($t$) for a dead load is,

$$t = \cdot 021\,S\,x\sqrt{\frac{y}{x}},$$

we have, when $x = 1{\cdot}25$, and $y = 3{\cdot}5 + {\cdot}002\,\mathrm{S}$,

$$t = {\cdot}026\,\mathrm{S}\sqrt{2{\cdot}8 + {\cdot}0016\,\mathrm{S}},$$

and $$t_1 = \frac{2\,t\,\mathrm{T}}{\mathrm{T} + t},$$

which equation, taking the limiting strain at 80 cwt. per square inch as before, gives the following results:

| Span in feet. | Strain in cwts. per sq. in. | Depth. |
|---|---|---|
| 300 | 23·8 | $\frac{1}{6}$ |
| 400 | 31 | |
| 500 | 37·7 | |
| 600 | 43·8 | |
| 700 | 48·6 | |
| 800 | 55 | |
| 900 | 60 | |
| 1000 | 64·9 | |
| 1100 | 69·3 | |
| 1200 | 73·5 | |
| 1300 | 77·4 | |
| 1400 | 81 | $\frac{1}{7}$ |

The value of the multiple $\frac{t_1}{\mathrm{T} - t_1}$, that is, the weight of iron in terms of the useful load, will be as follows:

| Span in feet. | | | Multiple. |
|---|---|---|---|
| 300 | $\frac{23{\cdot}8}{56{\cdot}2}$ | = | ·42 |
| 400 | $\frac{31}{49}$ | = | ·63 |
| 500 | $\frac{37{\cdot}7}{42{\cdot}3}$ | = | ·9 |
| 600 | $\frac{43{\cdot}8}{36{\cdot}2}$ | = | 1·22 |
| 700 | $\frac{48{\cdot}6}{31{\cdot}4}$ | = | 1·55 |
| 800 | $\frac{55}{25}$ | = | 2·2 |
| 900 | $\frac{60}{20}$ | = | 3 |
| 1000 | $\frac{64{\cdot}9}{15{\cdot}1}$ | = | 4 3 |

| Span in feet. | | | Multiple. |
|---|---|---|---|
| 1100 | $\frac{69 \cdot 3}{10 \cdot 7}$ | = | 6·5 |
| 1200 | $\frac{73 \cdot 5}{6 \cdot 5}$ | = | 11·3 |
| 1300 | $\frac{77 \cdot 4}{2 \cdot 6}$ | = | 30 |
| 1400 | | = | ∞ |

### Type 9.—*Suspension with Stiffening Girder.*

The combined lightness and sustaining power of a festoon of rope stretched between two supports, could hardly have failed to attract, at a very early date, the attention of thoughtful practical men. The distance apart at which the points of support might be placed, would appear almost infinite as compared with what would have been the limit had the intervening space been spanned by a solid bar of the same size as the rope, merely resting on the supports. When, therefore, the occasion to throw a light structure across a wide river or ravine first arose, the similarity of the conditions to the case of the rope with its two distant points of support must almost necessarily have suggested a similar mode of procedure; and knowing the great superiority of the tensional strength of iron over rope, it was only natural that the "suspension bridge," in its simplest form, should be evolved, and that it should be the earliest form of the "long-span bridge," as understood in our definition.

The great instability, however, of this mode of construction was at first inadequately appreciated. Numerous failures had occurred before it began to be clearly understood that a moderate force, applied at regular intervals, would produce an isochronous movement of sufficient extent to effect at last the destruction of the structure. The first modification suggested by these accidents was the insertion of check ties, or even inverted suspension chains, to hold down the platform, and so prevent its oscillation. This was but a mere empirical remedy, and a more complete and scientific investigation of the conditions of a suspension bridge was imperative before a successful result could be obtained. As the problem presented no important difficulties, the solution of it was delayed an unreasonable length of time.

A little consideration will show that, to secure the stability of a well-proportioned suspension bridge, it is only necessary to protect the chains from any considerable change of form, and the consequent tremor and oscillation. To comply with this condition, the forces acting on the chains must maintain always the same direction and the same relative proportions to one another, but it is not essential to the conditions of equilibrium that the *amount* of these strains should remain constant. This being so, it follows that if the actual distribution of the load, or some other conditions, be such that the relative proportions of the forces would not be maintained at all times, sufficient transverse strength must be provided in the structure to effect the required distribution of the forces acting on the chain. It matters little where this strength be supplied; in some few instances it has been obtained by bracing together two sets of chains, one lying under the other, thus assimilating the chains to an inverted arched rib; in fact, a plate girder section for the suspension member is the subject of a patent. Ordinarily, the required transverse strength is obtained in the construction of the platform, where it is equally efficient, and rather adds to, than detracts from, the elegance of the structure.

Although by this arrangement we effectually counteract the dangerous isochronous movement, yet, as the economic depth of the stiffening girder is comparatively very small, and as a double wave of deflection precedes and follows the load as it rolls over the bridge, a considerable amount of vibration must necessarily still exist, although not dangerous in its effects. This objectionable peculiarity of the ordinary suspension bridge is probably the cause of its almost universal condemnation as unfit for railway purposes.

We believe, nevertheless, that the bad proportions of the earlier bridges, and the consequent failures, have created an unwarranted prejudice against the system. There is no theoretical or practical reason why a suspension bridge should not be made of any required degree of rigidity; but whether this could be done *economically* remains to be seen. Whatever can be effected on the principle of the arch may also be obtained in a suspension bridge. Thus, if we were to invert our last type, the arched rib with braced spandrils, jointed at three points, we should obtain a rigid suspension bridge, free to expand and contract under changes of temperature. Again, provided we supply adequate transverse strength to the two halves of the bridge, it is immaterial what form our bracing may assume. As, however, we must have a horizontal girder at the level of the platform, it is more convenient to truss between that member and the arched ribs or chains, as the case may be, than to insert a special member.

It is obvious, then, that it is possible to design an immense variety of forms of rigid suspension bridges; the most elementary type being probably a couple of inclined straight beams, the outer ends of which are attached to the top of the piers, and the near ends jointed together. All such bridges may be referred to our last type, and the other systems will be included in our remaining types. At present we shall confine our attention to the ordinary suspension bridge with stiffening platform girder.

### *Chains and Suspending Bars.*

In order to obtain great stiffness in the suspending bars and other portions of the structure, and so fit it for its duty of carrying a heavy rolling load with little vibration, we shall provide a mass of metal for the construction of the suspension portion of the bridge, exclusive of the stiffening girder, equal to the amount we have found necessary in an arched bridge carrying an entirely dead load. The strain per square inch will therefore be the same in both instances:

$$t = \cdot 021\, S\, x \sqrt{\frac{y}{x}}$$

where $x = 1{\cdot}25$, and $y = 3{\cdot}2 + \cdot 002\, S$.

### *Lattice Stiffening Girder.*

Eliminating the complicating elements the unequal deflections of the chain and girder introduce into the question, by assuming a certain amount of preliminary adjustment to be effected during the erection of the bridge, we find the maximum bending moment on the stiffening girder takes place when the bridge is two-thirds loaded, at which time it will, in terms of that on the entire bridge, be equal to $\frac{1}{3}\left(\frac{2}{3}\right)^2 = \frac{4}{27}$. Therefore, $w$ being the load in cwts. per foot run, and T the strain in cwts. per square inch, the required sectional area of flange will be:

$$a = \frac{S^2 w}{54\, d\, T}.$$

But, since all parts of the girder are successively exposed to nearly the same amount of strain the theoretical mass of the web of the stiffening girder will be about $1\frac{1}{2}$ times that of a similar ordinary girder; therefore economic depth $= \frac{S x}{3 y}$. Hence, since weight per foot run of the girder in cwts. (W) equals ·03 cwt. $\times 4\, a\, x$, we have:

$$W = \frac{S w y}{150\, T}.$$

Putting the rolling load $w = 25$ cwt. per foot, the gross useful load $= 32$ cwt. per foot, and $T = 80$ cwt. per square inch, the weight of the stiffening girder, in terms of the useful load, will be $= \frac{S y}{15000}$; and the total load on the chains $= 1 + \frac{S y}{15000}$.

The strain in cwts. per square inch on the chains due to the suspension portion of the bridge will be,

$$t = \cdot 026\, S \sqrt{2 \cdot 56 + \cdot 0016\, S},$$

which equation gives the following results:

| Span in feet. | Strain in cwts. per sq. in. | Depth of Stiffening Girder. | Deflection of Chains. |
|---|---|---|---|
| 300 | 13·5 | $\frac{1}{18}$ | $\frac{1}{5}$ |
| 400 | 18·6 | | |
| 500 | 24 | | |
| 600 | 29·4 | | |
| 700 | 35 | | |
| 800 | 40·8 | | |
| 900 | 46·8 | | |
| 1000 | 53 | | |
| 1100 | 59·4 | | |
| 1200 | 66 | | |
| 1300 | 73 | | |
| 1400 | 80 | $\frac{1}{27}$ | $\frac{1}{6}$ |

The total weight of iron in the suspension portion of the bridge and in the stiffening girder will be expressed, in terms of the useful load, by the equation:

$$\text{Multiple} = \frac{\mathrm{S}\,y}{15000} + \left\{\frac{t}{\mathrm{T} - t}\left(1 + \frac{\mathrm{S}\,y}{15000}\right)\right\}.$$

Taking $y = \cdot 5 + \cdot 0035\,\mathrm{S}$, the results are as follow:

| Span in feet. | $\frac{\mathrm{S}\,y}{15000}$. | Multiple. |
|---|---|---|
| 300 | ·12 | ·36 |
| 400 | ·17 | ·53 |
| 500 | ·22 | ·76 |
| 600 | ·27 | 1·02 |
| 700 | ·33 | 1·4 |
| 800 | ·39 | 1·9 |
| 900 | ·46 | 2·6 |
| 1000 | ·53 | 3·5 |
| 1100 | ·61 | 5·4 |
| 1200 | ·68 | 8·7 |
| 1300 | ·77 | 19·4 |
| 1400 | | ∞ |

### Type 10.—*Suspended Girder.*

A perfect suspension bridge would be a structure combining the rigidity of a girder with the lightness characteristic of the former system. In attempting to arrive at this desideratum, the question naturally suggests itself, what is the fundamental difference between the two systems, giving them these attributes of rigidity and lightness respectively? Now, the girder is rigid because the depth of bracing, and consequently the resistance offered to change of form, is comparatively large; and the suspension bridge is light, because the compression member of the girder is dispensed with; for although an equivalent resistance must be supplied elsewhere in the land chains and anchorage, yet the mass of metal so employed does not add to the load on the bridge, as it would have done had it been in the form of a compression member.

Let us imagine an inverted bowstring girder, the top compression member being straight, and braced to the curved tension member in the usual manner, then it must be granted that this girder will present equal rigidity with the bowstring type, and that it will also require about the same mass of metal in its construction. But the problem to be solved is how to reduce the mass without impairing the rigidity; and, from what we have already observed, it will be seen that this can only be effected by transferring a portion, or the whole, of the metal required in the top member to some other point where it will be equally efficacious, and will not add to the load on the girder.

Now, the strain on the top member of the girder is compression (+), and it is obvious that if we can put an initial tension (−) on that member, the resulting strain to be provided for will be the difference between those strains, and the mass of metal required will be proportional to that difference. We shall now show how, by a very simple contrivance, we shall be enabled to put any required degree of initial tension on the top member of our girder.

Let one end of the girder be made fast to its pier, and let the other end, instead of resting immediately on the pier, be suspended by an inclined link from it; then, by the resolution of forces, it follows that the initial tension on the top member will bear the same ratio to the entire weight of the bridge as the horizontal component of the inclination of the link bears to double the vertical component. It follows from this that if the inclination of the link be tangential to the curve of equilibrium, due to the load, the initial tension will just neutralize the final compression. As, however, the direction of the tangent to the curve of equilibrium varies with the position of the rolling load, whilst the inclination of the link necessarily remains constant, we shall have to reserve a certain proportion of the compression member to meet the consequent strains. We have, nevertheless, by this arrangement, disposed of the great mass of the compression member without impairing the rigidity or the freedom of movement under changes of temperature, pertaining to the ordinary bowstring girder.

The "suspended girder," as, for want of a better name, we have christened the foregoing type, was, to the best of our belief, first introduced by Mr. Fowler, who proposed to construct his 750 ft. span high-level bridge over the Thames on that principle. There can be no doubt that, had that structure been carried out, the great depth of bracing maintained at the centre of the span—the point of maximum deflection—would have secured for it almost perfect immunity from those vibrating impulses which, although not appearing in theoretical calculations, manifest themselves in a very palpable manner in ordinary "rigid" suspension bridges.

### *Dead Load.*

With a dead load uniformly distributed, the mass of metal required will be similar to that employed in the suspension portion of our last type. The strain in cwts. per square inch due to the weight of the structure itself will therefore be:

$$t = \cdot 03\left(\cdot 71\, \mathrm{S}\, x \sqrt{\frac{y}{x}}\right).$$

### *Rolling Load.*

The mass required in the suspension portion will be the same as that for a dead load $= a\left(\mathrm{S} + \frac{7\, d^2}{\mathrm{S}}\right)$. That of the bracing will be identical with the corresponding members of the bowstring girder $= a\left(\frac{14\, d^2}{\mathrm{S}} + \frac{\mathrm{S}}{5}\right)$. The mass of the compression member necessary to be retained, in order to provide for the strains resulting from the unequal distribution of the rolling load, will be $\cdot 27\, a\, \mathrm{S}$. Taking $x$ and $y$ coefficients as before, the total mass for each square inch sectional area of chains will be:

$$1\cdot 47\, \mathrm{S}\, x + \frac{21\, d^2\, y}{\mathrm{S}}.$$

But mass × $\frac{1}{8}$ span × ·03 cwt. equals the moment in cwts.

$$\mu = \cdot 03\,(\cdot 18\,\mathrm{S}\,x + 2\cdot 62\,d^2\,y).$$

Now economic $d^2 = \frac{1\cdot 47\,\mathrm{S}\,x}{21\,y}$; therefore $d = \frac{10}{38}\mathrm{S}\sqrt{\frac{x}{y}}$; and the strain in cwts. per square inch $= \frac{\mu}{d}$, will be:

$$t = \cdot 03\left(1\cdot 4\,\mathrm{S}\,x\sqrt{\frac{y}{x}}\right).$$

### *Mixed Load.*

Assuming, as in previous instances, the mixed load to be composed of $\frac{3}{4}$ rolling and $\frac{1}{4}$ dead load, the mean coefficient will be $1{\cdot}4 \times \frac{3}{4} + {\cdot}71 \times \frac{1}{4} = 1{\cdot}23$; or, taking the coefficient ·71 as the unit of measurement, that for the mixed load will be $\frac{1\cdot 23}{\cdot 71} = 1{\cdot}73$ times that amount. Hence the strain in cwts. per square inch due to the dead load being,

$$t = \cdot 021\,\mathrm{S}\,x\sqrt{\frac{y}{x}},$$

we have, when $x = 1\cdot 25$ and $y = 3\cdot 2 + \cdot 002\,\mathrm{S}$,

$$t = \cdot 026\,\mathrm{S}\sqrt{2\cdot 8 + \cdot 0016\,\mathrm{S}},$$

and $$t_1 = \frac{1\cdot 73\,t\,\mathrm{T}}{\mathrm{T} + \cdot 73\,t},$$

which equation gives the following results:

| Span in feet. | Strain in cwts. per sq. in. | Depth. |
|---|---|---|
| 300 | 21 | $\frac{1}{7}$ |
| 400 | 27·5 | |
| 500 | 34 | |

| Span in feet. | Strain in cwts. per sq. in. | Depth. |
|---|---|---|
| 600 | 40 | |
| 700 | 46 | |
| 800 | 51·5 | |
| 900 | 57 | |
| 1000 | 62 | |
| 1100 | 67 | |
| 1200 | 71·5 | |
| 1300 | 76 | |
| 1400 | 80 | $\frac{1}{8}$ |

The multiple $\frac{t_1'}{T - t_1}$ will have the following values:

| Span in feet. | | | Multiple. |
|---|---|---|---|
| 300 | $\frac{21}{59}$ | = | ·36 |
| 400 | $\frac{57 \cdot 2}{52 \cdot 5}$ | = | ·52 |
| 500 | $\frac{34}{46}$ | = | ·74 |
| 600 | $\frac{40}{40}$ | = | 1·00 |
| 700 | $\frac{46}{34}$ | = | 1·35 |
| 800 | $\frac{51 \cdot 5}{28 \cdot 5}$ | = | 1·82 |
| 900 | $\frac{57}{23}$ | = | 2·48 |
| 1000 | $\frac{62}{18}$ | = | 3·38 |
| 1100 | $\frac{67}{13}$ | = | 5·16 |
| 1200 | $\frac{71 \cdot 5}{8 \cdot 5}$ | = | 8·40 |
| 1300 | $\frac{74}{4}$ | = | 19·00 |
| 1400 | | = | $\infty$ |

### Type 11.—*Straight-Link Suspension.*

We have found the mass of our last type girder to be similar to that of the bowstring, with a large proportion of the compression member removed; and in the same manner it can be shown that our present type will be analogous to that of the straight-link girder, with the *whole* of the compression member, together with a certain proportion of the mass of the verticals affected thereby taken away. All our remarks, with reference to the deflection and the conditions of the straight-link girder, will, *mutatis mutandis*, apply equally to the straight-link suspension; consequently, it will be merely necessary here to effect the requisite modifications in our former calculations.

### *Rolling Load.*

The mass of the ties will be the same as before, $= a\left(\mathrm{S}\,x + \frac{21\,d^2 y}{4\,\mathrm{S}}\right)$. But mass × $\frac{1}{6}$ span × ·03 cwt. = moment in cwts.: therefore, for 1 square inch,

$$\mu = \cdot 03\left(\frac{\mathrm{S}^2 x}{6} + \frac{7\,d^2 y}{8}\right).$$

Now $\frac{\mu}{d}$ = strain in cwts, per square inch; and since economic $d^2 = \frac{4\,\mathrm{S}^2 x}{21\,y}$, we have $d = \frac{\mathrm{S}}{2\cdot 3}\sqrt{\frac{x}{y}}$; and the strain

$$t = \cdot 03\left(\cdot 76\,\mathrm{S}\,x\sqrt{\frac{y}{x}}\right).$$

### *Dead Load.*

The mass of the ties will be the same as for the rolling load $= a\left(\mathrm{S}\,x + \frac{21\,d^2 y}{4\,\mathrm{S}}\right)$. That of the verticals will be the additional amount of $a\left(\frac{13\,d^2 y}{4\,\mathrm{S}}\right)$. The total mass of the girder consequently will be:

$$a\left(\mathrm{S}\,x + \frac{17\,d^2 y}{2\,\mathrm{S}}\right).$$

But mass × $\frac{1}{6}$ span × ·03 cwt. = moment in cwts.; hence if $a = 1$ square inch,

$$\mu = \cdot 03\left(\frac{S^2 x}{6} + \frac{23 d^2 y}{16}\right).$$

Since economic $d^2 = \frac{2 S x}{17 y}$, we have $d = \frac{S}{2 \cdot 93}\sqrt{\frac{x}{y}}$; and the strain in cwts. per square inch equal to

$$t = \cdot 03\left(\cdot 98\, S x \sqrt{\frac{y}{x}}\right).$$

### *Mixed Load.*

When $x = 1 \cdot 25$, and $y = 3 \cdot 2 + \cdot 002\, S$, we have $t = \cdot 037\, S \sqrt{2 \cdot 56 + \cdot 0016\, S}$; and the strain in cwts. per square inch due to the weight of the straight-link suspension bridge for a railway will be:

$$t_1 = \frac{\cdot 78\, t\, T}{T - \cdot 22\, t}.$$

With the limiting strain T = 80 cwt., we obtain the following results:

| Span in feet. | Strain in cwts. per sq. in. | Depth. |
|---|---|---|
| 300 | 16 | 1 |
| 400 | 22 | |
| 500 | 28·6 | |
| 600 | 35·2 | |
| 700 | 43·8 | |
| 800 | 51·8 | |
| 900 | 61·7 | |
| 1000 | 71·6 | |
| 1100 | 82·5 | $\frac{1}{7}$ |

The weight ot iron in terms of the useful load will be as follows:

| Span in feet. | | | Multiple. |
|---|---|---|---|
| 300 | $\frac{16}{64}$ | = | ·25 |
| 400 | $\frac{22}{58}$ | = | ·38 |
| 500 | $\frac{28·6}{51·4}$ | = | ·56 |
| 600 | $\frac{35·2}{44·8}$ | = | ·78 |
| 700 | $\frac{43·8}{36·2}$ | = | 1·22 |
| 800 | $\frac{51·8}{28·2}$ | = | 1·85 |
| 900 | $\frac{61·7}{18·3}$ | = | 3·37 |
| 1000 | $\frac{71·5}{8·5}$ | = | 7·53 |
| 1100 | | = | ∞ |

## WEIGHT OF IRON IN TYPE BRIDGES.

Having thus ascertained the weight of iron, in terms of the "useful" load, required in the construction of each of our type girders, the first stage of our inquiry is brought to a termination. It will now be necessary to determine the specific amount of the load in each instance, as upon that depends the absolute weight of the main girders. It will then be necessary to ascertain the weight of iron required in each bridge, in addition to that of the main girders; for it is obvious that the respective advantages of the different systems can only be fairly tested by a comparison of the *gross* weight of iron required in the complete construction of the bridge in each instance. We shall not, therefore, waste space by giving a summary of the results in their present incomplete form, but proceed at once with the necessary steps for obtaining a final result.

Now, the gross "useful" load on the main girders will be made up of the following elements: 1st, the horizontal bracing necessary to ensure the lateral stability of the platform, &c.; 2nd, the girders carrying the platform, consisting of cross girders, with longitudinal bearers under each rail; 3rd, the platform covering, including the permanent way; and, 4th, the rolling load. The first two elements will obviously affect the gross weight of iron in the bridge in a double degree, for not only will their own proper weight appear in the sum-total, but it will also affect that amount indirectly, by imposing an additional load upon the main girders. As the joint weight of the bracing and the platform girders will be governed partly by the span and partly by the type of the main girders, it will be necessary to estimate the amount for each individual case.

In a double line of railway bridge of ordinary span, the weight of the platform girders will be about 7 cwt. per foot run of the bridge; and this amount will increase in a certain ratio with the span, as the width of platform assumed in the instance of the 300 ft. bridge would be wholly inadequate to secure lateral stability at the higher spans. Now, the maximum span, to which we shall have occasion to refer, will be about 3200 ft.; and in order to obtain the required degree of lateral stiffness, the effective depth of the horizontal bracing should be about 100 ft., and, consequently, the cross girder must be of that span, although possibly no greater length than 25 ft., at the centre, will be occupied by the railway proper. Assuming, then, for the 3200 ft. span, cross girders 100 ft. span and 30 ft. apart, with longitudinal girders of 30 ft. span under each rail, the weight of iron in the platform will be as follows:

**Total weight of the four girders under rails = $4\frac{2}{3}$ cwt. per foot run of bridge.**

Weight of cross girder = $5\frac{1}{2}$ cwt. per foot run.

$$\therefore \quad \frac{5\frac{1}{2} \times 100}{30} = 18\tfrac{1}{3} \text{ cwt. per foot run of bridge.}$$

The total weight of the platform girders will, therefore, be $4\frac{2}{3} + 18\frac{1}{3} = 23$ cwt. per foot run of the bridge.

The heaviest class of horizontal bracing required in any of our type constructions we shall assume to be equal to $\frac{3}{5}$ the weight of the preceding platform girders. The total weight of iron required, in addition to that of the main girders, will, therefore, be as follows:

300 ft. span, $1\frac{3}{5} \times 7 = 11$ cwt. per foot span.
3200 „ $1\frac{3}{5} \times 23 = 37$ „

Now, $11 : 37 :: \frac{2}{3}\sqrt{300} : \frac{2}{3}\sqrt{3200}$ (nearly); consequently, the weight of iron in the platform girders and the heaviest class of horizontal bracing will be expressed by the simple equation:

Weights in cwts. per foot run of bridge = $\frac{2}{3}\sqrt{\text{Span in feet.}}$

The remainder of the load on a long-span railway bridge may be considered as a constant amount for all spans, equal, for the double line, to 40 cwt. per foot run of the bridge. It follows, therefore, that the "useful" load in cwts. per foot run (L), to be carried by any girder, will be expressed generally by the equation:

$$L = 40 + x\sqrt{S.}$$

For the heaviest description of horizontal bracing, we have found the value of $x$ to be $\frac{2}{3}$. Substituting the proper values for the various required degrees of lateral strength, and classifying our type forms accordingly, we obtain the following equations for the "useful" load:

Types 1, 2, 3, 4 .. $L = 40 + \frac{2}{3}\sqrt{S}$.
„ 5, 6, 7, 8 .. $L = 40 + \frac{7}{12}\sqrt{S}$.
„ 9, 10, 11 .. $L = 40 + \frac{1}{2}\sqrt{S}$.

Now, W being the *gross* weight of iron in cwts. per foot span required in the construction of a bridge, and M the multiple arrived at in our previous investigations for the particular case under consideration, we have:

$$W = ML + L - 40.$$

Substituting the given values of M and L in this equation, we obtain the results shown in the following Tables:

### Type 1.—*Box Girders with Plate Webs.*

| Span in feet. | M. | L — 40. | W. |
|---|---|---|---|
| 300 | ·86 | 11·5 | 55 |
| 400 | 1·5 | 13·3 | 93 |
| 500 | 2·6 | 15 | 156 |
| 600 | 5·6 | 16·3 | 331 |
| 700 | 25 | 17·6 | 1458 |

### Type 2.—*Lattice Girders.*

| | | | |
|---|---|---|---|
| 300 | ·51 | 11·5 | 38 |
| 400 | ·87 | 13·3 | 60 |
| 500 | 1·5 | 15 | 96 |
| 600 | 2·83 | 16·3 | 179 |
| 700 | 8·3 | 17·8 | 496 |

### Type 3.—*Bowstring Girders.*

| | | | |
|---|---|---|---|
| 300 | ·51 | 11·5 | 37 |
| 400 | ·79 | 13·3 | 56 |
| 500 | 1·19 | 15 | 80 |
| 600 | 1·86 | 16·3 | 120 |
| 700 | 3 | 17·6 | 190 |
| 800 | 5·15 | 18·8 | 322 |
| 900 | 15 | 20 | 920 |

### Type 4.—*Straight Link and Boom.*

| Span in feet. | M. | L — 40. | W. |
|---|---|---|---|
| 300 | ·4 | 11·5 | 32 |
| 400 | ·72 | 13·5 | 52 |
| 500 | 1·28 | 15 | 102 |
| 600 | 2·61 | 16·3 | 164 |
| 700 | 9·97 | 17·6 | 593 |

### Type 5.—*Cantilever Lattice.*

| | | | |
|---|---|---|---|
| 300 | ·39 | 10·1 | 29 |
| 400 | ·61 | 11·7 | 43 |
| 500 | ·88 | 13·1 | 59 |
| 600 | 1·23 | 14·2 | 80 |
| 700 | 1·7 | 15·4 | 109 |
| 800 | 2·5 | 16·4 | 157 |
| 900 | 3·6 | 17·4 | 225 |
| 1000 | 5·43 | 18·4 | 334 |
| 1100 | 9·25 | 19·3 | 578 |
| 1200 | 21 | 20·3 | 1469 |

### Type 6.—*Cantilever Lattice, varying depth.*

| | | | |
|---|---|---|---|
| 300 | ·3 | 10·1 | 25 |
| 400 | ·46 | 11·7 | 35 |
| 500 | ·65 | 13·1 | 47 |
| 600 | ·88 | 14·2 | 62 |
| 700 | 1·2 | 15·4 | 82 |
| 800 | 1·61 | 16·4 | 107 |
| 900 | 2·17 | 17·4 | 142 |
| 1000 | 2·96 | 18·4 | 191 |
| 1100 | 4·15 | 19·3 | 265 |
| 1200 | 6·1 | 20·3 | 388 |
| 1300 | 11·2 | 21·1 | 705 |
| 1400 | 22 | 21·8 | 1380 |

### Type 7.—*Continuous Girder, varying depth.*

| | | | |
|---|---|---|---|
| 300 | ·3 | 10·1 | 25 |
| 400 | ·46 | 11·7 | 35 |
| 500 | ·65 | 13·1 | 47 |
| 600 | ·85 | 14·2 | 60 |
| 700 | 1·1 | 15·4 | 76 |

| Span in feet. | M. | L — 40. | W. |
|---|---|---|---|
| 800 | 1·33 | 16·4 | 91 |
| 900 | 1·61 | 17·4 | 110 |
| 1000 | 1·84 | 18·4 | 128 |
| 1100 | 2·1 | 19·3 | 144 |
| 1200 | 2·48 | 20·3 | 169 |
| 1300 | 2·87 | 21·1 | 199 |
| 1400 | 3·37 | 21·8 | 230 |
| 1500 | 3·91 | 22·6 | 267 |
| 1600 | 4·58 | 23·3 | 312 |
| 1700 | 5·35 | 24·1 | 366 |
| 1800 | 6·47 | 24·6 | 449 |
| 1900 | 7·98 | 25·3 | 546 |
| 2000 | 10·42 | 26·1 | 715 |
| 2100 | 14·32 | 26·6 | 981 |
| 2200 | 20 | 27·3 | 1375 |
| 2300 | 29 | 28 | 2000 |

### Type 8.—*Arched Ribs with Braced Spandrils.*

| | | | |
|---|---|---|---|
| 300 | ·42 | 10·1 | 36 |
| 400 | ·63 | 11·7 | 44 |
| 500 | ·9 | 13·1 | 60 |
| 600 | 1·22 | 14·2 | 80 |
| 700 | 1·55 | 15·4 | 100 |
| 800 | 2·2 | 16·4 | 135 |
| 900 | 3 | 17·4 | 189 |
| 1000 | 4·3 | 18·4 | 268 |
| 1100 | 6·5 | 19·3 | 408 |
| 1200 | 11·3 | 20·3 | 701 |
| 1300 | 30 | 21·1 | 1850 |

### Type 9.—*Suspension and Stiffening Girder*

| | | | |
|---|---|---|---|
| 300 | ·36 | 8·6 | 26 |
| 400 | ·53 | 10 | 36 |
| 500 | ·76 | 11·2 | 50 |
| 600 | 1·02 | 12·2 | 65 |
| 700 | 1·4 | 13·2 | 87 |
| 800 | 1·9 | 14·1 | 116 |
| 900 | 2·6 | 15 | 158 |
| 1000 | 3·5 | 15·8 | 211 |

| Span in feet. | M. | L — 40. | W. |
|---|---|---|---|
| 1100 | 5·4 | 16·6 | 322 |
| 1200 | 8·7 | 17·3 | 515 |
| 1300 | 19·4 | 18 | 1143 |

### Type 10.—*Suspended Girders.*

| | | | |
|---|---|---|---|
| 300 | ·36 | 8·6 | 26 |
| 400 | ·52 | 10 | 36 |
| 500 | ·74 | 11·2 | 49 |
| 600 | 1 | 12·2 | 64 |
| 700 | 1·35 | 13·2 | 85 |
| 800 | 1·82 | 14·1 | 113 |
| 900 | 2·48 | 15 | 152 |
| 1000 | 3·38 | 15·8 | 204 |
| 1100 | 5·16 | 16·6 | 308 |
| 1200 | 8·4 | 17·3 | 500 |
| 1300 | 19 | 18 | 1120 |

### Type 11.—*Straight-Link Suspension.*

| | | | |
|---|---|---|---|
| 300 | ·5 | 8·6 | 21 |
| 400 | ·38 | 10 | 30 |
| 500 | ·56 | 11·2 | 40 |
| 600 | ·78 | 12·2 | 53 |
| 700 | 1·22 | 13·2 | 78 |
| 800 | 1·85 | 14·1 | 114 |
| 900 | 3·37 | 15 | 200 |
| 1000 | 7·53 | 15·8 | 436 |

We have now before us the gross weight of iron per foot span required in the construction of bridges of every class of design, not absolutely eccentric, as we maintain those structures must be which cannot be referred to one or the other of the preceding types.

The weight per foot of the main span will not in itself represent the comparative costs of different types of superstructure even, and much less, as we have shown at the commencement of these papers, that of the entire bridge. With the exception of the first four, all our type constructions are dependent upon extraneous aid for their stability, and the proportional cost of that portion spread over the whole span must obviously be included in the proper weight of the span itself before a comparison can be made with either of the independent girders, where such provision is unnecessary. Thus, given a viaduct of three spans, the centre opening being double the others, then the average weight per foot of the superstructure, if constructed with independent girders, would obviously be the mean weight per foot of the large and small spans; whereas, should some other system, such as the cantilever be adopted, the average weight per foot would be that due to the long span throughout. We shall defer for the present, however, this branch of our inquiry, and proceed at once with the adaptation of our formulæ to steel structures.

## WEIGHT OF STEEL IN TYPE BRIDGES.

As we do not purpose making a digression on the subject of the comparative strength and other advantages of steel, we shall have little more to present than the tabular results given by the formulæ, which will be but a slightly modified form of those already advanced for similar iron structures.

The general conditions of the several types will be little influenced by the substitution of the new material. The deflections will be proportionally the same, although, of course, the specific amounts will be greater on account of the higher value of the limiting strain. The comparative weights of metal required in the construction of steel and iron long-span bridges will vary in a much higher ratio than the mere ultimate resistances of the two materials. Thus, if the strain due to the weight of the girder itself be 50 cwt. per square inch, there will, with a limiting strain of 80 cwt. per square inch, remain $80-50=30$ cwt. per square inch available for the "useful" load; whereas, had the limiting strain been 130 cwt., the residue would have been $130-50=80$ cwt. per square inch; consequently in such an instance, although the ultimate resistances of the two materials are only as 13 : 8, the practical available strengths would be as 8 : 3, or about $1\frac{2}{3}$ times the amount which might at first sight appear. We shall consider a strain of 130 cwt. per square inch on steel to represent the same factor of safety as a strain of 80 cwt. per square inch on iron. This will be well within the limits, and we take it purposely so, to avoid all exaggeration of the probable advantages accruing to steel as the material for long-span bridges. We shall not make any special modification in our formulæ on account of the slightly increased specific weight of steel, but include all necessary changes in the altered values given to the coefficient $y$. It will be unnecessary to give the process in the same detail as before; we shall merely give the final formulæ and the combined Tables for each type.

### Type 1.—*Box Girders with Plate Webs* (*Steel*).

As the same causes which make this a disadvantageous form for a long-span *iron* bridge operate with still greater effect when *steel* is in question, we do not consider it necessary to investigate the conditions of this type in the present instance.

### Type 2.—*Lattice Girders* (*Steel*).

The weight of the platform girders and bracing in *steel* we shall take to be $\frac{2}{3}$ of their former amount, but we shall in all instances add five per cent. for contingencies to the value of W as given by the formulæ. The notation will remain as before; that is, S=span in feet; T = limiting strain=130 cwt. per square inch; $x$ and $y$ coefficients depending upon the practical construction of the girders; $t_1$ the strain in hundred-weights per square inch resulting from the load of the structure itself; M the weight of iron in terms of the "useful" load (L); and W the gross weight of iron per foot run of the entire bridge.

$t_1 = {\cdot}03\,S y.$ When $y = 2{\cdot}7 + {\cdot}001\,S$, we have:

$$t_1 = {\cdot}105\,S + {\cdot}00003\,S^2.$$

$$M = \frac{t_1}{T - t_1}. \quad L = 40 + \frac{4}{9}\sqrt{S}.$$

$$W = ML + L - 40.$$

| Span in feet. | $t_1$ | M. | W. |
|---|---|---|---|
| 300 | 34·2 | ·35 | 25 |
| 400 | 46·8 | ·56 | 38 |
| 500 | 60 | ·8 | 52 |
| 600 | 74·8 | 1·35 | 83 |
| 700 | 88·2 | 2·11 | 126 |
| 800 | 103·2 | 3·85 | 225 |
| 900 | 118·8 | 10·6 | 607 |
| 1000. | 135 | ∞ | ∞ |

## Type 3.—*Bowstring Girder* (*Steel*).

$$t_1 = \frac{1{\cdot}44\,t\,T}{T + {\cdot}44\,t}.$$

When $y = 4{\cdot}2 + {\cdot}002\,S$, we have $t = {\cdot}04\,S\,\sqrt{3{\cdot}36 + {\cdot}0016\,S}.$

$$M = \frac{t_1}{T - t_1}. \quad L = 40 + \frac{4}{9}\sqrt{S}.$$

$$W = ML + L - 40.$$

| Span in feet. | $t_1$ | M. | W. |
|---|---|---|---|
| 300 | 31 | ·31 | 24 |
| 400 | 41·5 | ·46 | 33 |
| 500 | 51·5 | ·65 | 45 |
| 600 | 61·5 | ·89 | 59 |
| 700 | 71 | 1·2 | 78 |
| 800 | 80·5 | 1·62 | 102 |
| 900 | 90 | 2·25 | 140 |
| 1000 | 98 | 3 | 184 |
| 1100 | 103·5 | 3·9 | 238 |
| 1200 | 115·5 | 8 | 480 |
| 1300 | 123 | 17·5 | 1047 |
| 1400 | 131 | ∞ | ∞ |

## Type 4.—*Straight-Link Girder* (*Steel*).

$$t_1 = \frac{\cdot 7\, t\, T}{T - \cdot 3\, t}.$$

When $y = 4\cdot 2 + \cdot 002\,S$, we have $t = \cdot 059\,S\sqrt{3\cdot 36 + \cdot 0016\,S}$.

$$M = \frac{t_1}{T - t_1}. \quad L = 40 + \frac{4}{9}\sqrt{S}.$$

$$W = M L + L - 40.$$

| | | | |
|---|---|---|---|
| 300 | 25·4 | ·24 | 21 |
| 400 | 35·8 | ·38 | 29 |
| 500 | 46·2 | ·55 | 40 |
| 600 | 58·8 | ·83 | 55 |
| 700 | 71·5 | 1·22 | 78 |
| 800 | 86·5 | 1·99 | 123 |
| 900 | 101·6 | 3·58 | 214 |
| 1000 | 124·6 | 23·1 | 1304 |
| 1100 | 140·7 | ∞ | ∞ |

## Type 5.—*Cantilever Lattice, uniform Depth* (*Steel*).

$t_1 = \sqrt{26\,S + \cdot 008\,S^2 + 32000} - 179$. When $y = 3\cdot 3 + \cdot 001\,S$,

$$M = \frac{t_1 b}{8(T - t_1)}. \quad L = 40 + \frac{7}{18}\sqrt{S}.$$

$$W = M L + L - 40.$$

| Span in feet. | $t_1$ | M. | W. |
|---|---|---|---|
| 300 | 22 | ·29 | 22 |
| 400 | 30 | ·45 | 31 |
| 500 | 38 | ·62 | 42 |
| 600 | 46 | ·86 | 54 |
| 700 | 54 | 1·13 | 70 |
| 800 | 63 | 1·5 | 91 |
| 900 | 70 | 1·93 | 116 |
| 1000 | 78 | 2·53 | 152 |
| 1100 | 86 | 3·34 | 198 |
| 1200 | 95 | 4·74 | 280 |
| 1300 | 103 | 6·76 | 397 |
| 1400 | 111 | 11·1 | 650 |
| 1500 | 120 | 22 | 1288 |
| 1600 | 128 | 120 | 7000 |
| 1700 | 137 | ∞ | ∞ |

### Type 6.—*Cantilever varying Economic Depth* (*Steel*).

$$t_1 = \sqrt{21\,S + \cdot006\,S^2 + 3200} - 179.$$

$$M = \frac{t_1 b}{8\,(T - t_1)}\cdot \quad L = 40 + \frac{7}{18}\sqrt{S}.$$

$$W = M\,L + L - 40.$$

| | | | |
|---|---|---|---|
| 300 | 18 | ·23 | 19 |
| 400 | 24 | ·32 | 24 |
| 500 | 31 | ·47 | 33 |
| 600 | 37 | ·61 | 41 |
| 700 | 44 | ·79 | 48 |
| 800 | 50 | 1·0 | 64 |
| 900 | 58 | 1·27 | 76 |
| 1000 | 64 | 1·57 | 98 |
| 1100 | 71 | 1·98 | 124 |
| 1200 | 79 | 2·6 | 158 |
| 1300 | 85 | 3·23 | 195 |
| 1400 | 91 | 4 | 242 |
| 1500 | 98 | 5·35 | 315 |
| 1600 | 106 | 7·84 | 481 |
| 1700 | 113 | 12·0 | 720 |
| 1800 | 120 | 22 | 1326 |
| 1900 | 128 | 120 | 7150 |
| 2000 | 134 | ∞ | ∞ |

## Type 7.—*Continuous Girder, varying Economic Depth (Steel).*

$$M = \frac{an + 3mb}{a + b} - 1. \quad L = 40 + \frac{7}{18}\sqrt{S}.$$

$$W = ML + L - 40.$$

| Span in feet. | M. | W. |
|---|---|---|
| 300 | ·23 | 19 |
| 400 | ·32 | 24 |
| 500 | ·47 | 33 |
| 600 | ·61 | 41 |
| 700 | ·79 | 48 |
| 800 | ·97 | 63 |
| 900 | 1·15 | 75 |
| 1000 | 1·3 | 84 |
| 1100 | 1·5 | 96 |
| 1200 | 1·7 | 109 |
| 1300 | 1·91 | 123 |
| 1400 | 2·12 | 137 |
| 1500 | 2·37 | 152 |
| 1600 | 2·63 | 168 |
| 1700 | 2·94 | 189 |
| 1800 | 3·26 | 210 |
| 1900 | 3·67 | 237 |
| 2000 | 4·08 | 264 |
| 2100 | 4·6 | 297 |
| 2200 | 5·13 | 332 |
| 2300 | 5·93 | 385 |
| 2400 | 6·7 | 434 |
| 2500 | 7·8 | 506 |
| 2600 | 9 | 585 |
| 2700 | 11 | 716 |
| 2800 | 13·2 | 861 |
| 2900 | 17·5 | 1141 |
| 3000 | 24·9 | 1625 |
| 3100 | 52 | 2821 |
| 3200 | 11·8 | 7000 |
| 4000 | ∞ | ∞ |

## Type 8.—*Arched Ribs with Braced Spandrils* (*Steel*).

$$t_1 = \frac{2\,t\,T}{T+t}.$$

When $y = 4\cdot5 + \cdot002\,S$, we have $t = \cdot026\,S\sqrt{3\cdot6+\cdot0016\,S}$.

$$M = \frac{t_1}{T-t_1}. \quad L = 40 + \frac{7}{18}\sqrt{S}.$$

$$W = M\,L + L - 40.$$

| Span in feet. | $t_1$ | M. | W. |
|---|---|---|---|
| 300 | 27·6 | ·27 | 21 |
| 400 | 36·8 | ·39 | 28 |
| 500 | 44·8 | ·52 | 36 |
| 600 | 53·2 | ·69 | 46 |
| 700 | 58·5 | ·87 | 57 |
| 800 | 68 | 1·1 | 71 |
| 900 | 74·8 | 1·35 | 84 |
| 1000 | 81·5 | 1·68 | 102 |
| 1100 | 87 | 2·02 | 125 |
| 1200 | 94·5 | 2·66 | 163 |
| 1300 | 100 | 3·33 | 203 |
| 1400 | 105 | 4·2 | 255 |
| 1500 | 110 | 5·5 | 333 |
| 1600 | 116 | 8·3 | 500 |
| 1700 | 120 | 12·2 | 735 |
| 1800 | 124 | 20·7 | 1240 |
| 1900 | 128 | 64 | 3836 |
| 2000 | 133 | ∞ | ∞ |

## Type 9.—*Suspension with Stiffening Girder* (*Steel*).

$$t_1 = \cdot026\,S\sqrt{3\cdot36 + \cdot0016\,S}.$$

When $y = 4\cdot2 + \cdot002\,S$. $L = 40 + \frac{1}{3}\sqrt{S}$.

$$M = \frac{S\,y_1}{20000} + \left\{\frac{t}{T-t}\left(1 + \frac{S\,y}{20000}\right)\right\}. \quad \text{When } y_1 = 5 + \cdot003\,S,$$

$$W = M\,L + L - 40.$$

| | | | |
|---|---|---|---|
| 300 | 15·3 | ·23 | 17 |
| 400 | 20·8 | ·34 | 24 |
| 500 | 26·5 | ·45 | 31 |
| 600 | 32·4 | ·6 | 39 |
| 700 | 38·5 | ·77 | 49 |

| Span in feet. | $t_1$ | M. | W. |
|---|---|---|---|
| 800 | 44·8 | ·97 | 60 |
| 900 | 51·3 | 1·22 | 74 |
| 1000 | 58 | 1·52 | 91 |
| 1100 | 64·9 | 1·9 | 113 |
| 1200 | 72 | 2·38 | 148 |
| 1300 | 79 | 2·98 | 175 |
| 1400 | 87 | 3·92 | 228 |
| 1500 | 93 | 5 | 289 |
| 1600 | 101 | 6·98 | 402 |
| 1700 | 109 | 10·51 | 600 |
| 1800 | 117 | 18·34 | 1055 |
| 1900 | 125 | 51·5 | 2910 |
| 2000 | 134 | ∞ | ∞ |

## Type 10.—*Suspended Girder* (*Steel*).

$$t_1 = \frac{1\cdot73\,t\,T}{T + \cdot73\,t}.$$

When $y = 4\cdot2 + \cdot002\,S$, we have $t = \cdot026\,S\sqrt{3\cdot36 + \cdot0016\,S}$.

$$M = \frac{t_1}{T - t_1}. \quad L = 40 + \tfrac{1}{3}\sqrt{S}.$$

$$W = M\,L + L - 60.$$

| | | | |
|---|---|---|---|
| 300 | 24·4 | ·23 | 17 |
| 400 | 31·6 | ·32 | 22 |
| 500 | 39·7 | ·43 | 28 |
| 600 | 47·5 | ·57 | 36 |
| 700 | 53·7 | ·72 | 44 |
| 800 | 64·5 | ·9 | 54 |
| 900 | 69 | 1·13 | 66 |
| 1000 | 75·5 | 1·4 | 81 |
| 1100 | 82·5 | 1·74 | 102 |
| 1200 | 88 | 2·14 | 124 |
| 1300 | 95 | 2·7 | 152 |
| 1400 | 101 | 3·3 | 186 |
| 1500 | 105 | 4·2 | 234 |
| 1600 | 110 | 5·5 | 307 |
| 1700 | 117 | 9 | 500 |
| 1800 | 123 | 17·5 | 961 |
| 1900 | 127 | 42·3 | 2320 |
| 2000 | 132 | ∞ | ∞ |

### Type 11.—*Straight-Link Suspension* (*Steel*).

$$t_1 = \frac{\cdot 78\, t\, \mathrm{T}}{\mathrm{T} - \cdot 22\, t}.$$

When $y = 4\cdot 2 + \cdot 002\,\mathrm{S}$, we have $t = \cdot 037\,\mathrm{S}\,\sqrt{3\cdot 36 + \cdot 0016\,\mathrm{S}}$.

$$\mathrm{M} = \frac{t_1}{\mathrm{T} - t_1}. \quad \mathrm{L} = 40 + \tfrac{1}{3}\sqrt{\mathrm{S}}.$$

$$\mathrm{W} = \mathrm{M\,L} + 40 - \mathrm{L}.$$

| Span in feet. | $t_1$ | M. | W. |
|---|---|---|---|
| 300 | 17·8 | ·16 | 14 |
| 400 | 24·2 | ·23 | 18 |
| 500 | 31·5 | ·3 | 23 |
| 600 | 39·4 | ·43 | 30 |
| 700 | 48 | ·58 | 38 |
| 800 | 56·5 | ·77 | 49 |
| 900 | 64·6 | ·96 | 60 |
| 1000 | 73 | 1·28 | 78 |
| 1100 | 84·5 | 1·86 | 110 |
| 1200 | 95·5 | 2·77 | 160 |
| 1300 | 109 | 5·2 | 368 |
| 1400 | 118·8 | 10·6 | 597 |
| 1500 | 129·5 | 359 | 2000 |
| 1600 | | | ∞ |

## SUMMARY.

The results of our various investigations are shown collectively in a graphical form in diagrams Nos. 1 and 2, the curved lines of which are obtained by plotting the gross weight of metal in cwts. per foot span, given in the final Tables for each type, to the vertical scale of 100 cwt. to the inch. A careful inspection of these diagrams will enable us easily to trace the comparative merits of the respective systems as far as the superstructure of the main span itself is involved, and to note the varying influence of the span in each instance.

A glance will be sufficient to assure us that the several types do not maintain the same relative economic positions throughout, since in many instances the lines cross one another, showing that, at the span corresponding to the point of intersection on the diagram, the weight of metal required in the construction of a bridge on either of the systems in question will be identical.

Briefly summarizing the results exhibited in the diagram for iron structures, we find that at the span of 300 ft., Type 11—the straight-link suspension bridge—obtains an advantage of some 20 per cent. over any other system, and that it maintains a certain advantage of diminishing value up to 700 ft. span, when it has to resign the lead to Type 7—the continuous girder of varying depth—which type maintains a rapidly increasing advantage over all others up to the limiting span. These two forms of construction, then, within their own proper spheres, appear to be the most economical possible, as regards the superstructure of the main span. It is obviously quite possible that in many instances anchorage could not be obtained for the suspension bridge, except at a cost which would render even our heaviest type—the box girder—a more economical form of construction.

The system ranking second in the scale of economy is Type 6, the cantilever lattice girder of varying depth, which maintains its relative position throughout, unaffected by the specific length of span. Types 9 and 10, the suspension with stiffening girder, and the suspended girder, succeed the last-named one. Although palpably different both in principle and appearance, the respective weights are almost identical throughout, being, up to 700 ft. span, little different to the preceding type. We now come to Type 5—the cantilever lattice girder of uniform depth—following closely on the heels of the last two systems up to 600 ft. span, when it is superseded by Type 8—the arched rib with braced spandrils. The independent girders, as might fairly be expected, occupy the lowest place on the list, although at 300 ft. span Type 4—the straight-link girder—shows a slight advantage over the arch. Within the limits of 400 or 500 ft. span, the straight link is the most economic form of the independent girder; above that span the bowstring girder surpasses it. Types 2 and 1—the lattice and box girders—conclude the list.

We have already insisted that the order of economy thus exhibited holds good only with reference to the superstructure of the main span, and that it does not represent the comparative costs of complete structures on the different systems, for which, indeed, no general rule could possibly be given. Some of our types require loftier and, *cæteris paribus,* more expensive piers than others; thus the piers for a suspension bridge will be higher than those for an arch, and the land-chains required in the former system will be another cause of excess. Probably the fairest way of arriving approximately at the true relative economy of the different systems in ordinary cases will be by ascertaining the gross *average* weight of iron per foot run required in the construction of viaducts consisting of three spans, of which the side spans are one-half the opening of the centre one. Under these circumstances, with the exception of the independent girders, the average weight per foot run will be that due to the long span, whilst in the latter instances the smaller weight per foot of the side spans will reduce the *average* weight per foot of the entire structure, as may be seen by the comparison of the following Tables with former ones:

S = span of centre opening; $\frac{S}{2}$ = span of side openings.

W = average weight of iron in cwts. per foot run of bridge.

*Iron Girder Viaducts.*

| S. | Box.<br>W. | Lattice.<br>W. | Bowstring.<br>W. | Straight link.<br>W. |
|---|---|---|---|---|
| 300 | 44 | 32 | 31 | 27 |
| 400 | 66 | 45 | 43 | 39 |
| 500 | 102 | 65 | 57 | 65 |
| 600 | 193 | 109 | 79 | 98 |
| 700 | 766 | 273 | 118 | 318 |
| 800 | | | 189 | |
| 900 | | | 494 | |

*Steel Girder Viaducts.*

| S. | Lattice.<br>W. | Bowstring.<br>W. | Straight link.<br>W. |
|---|---|---|---|
| 300 | 21 | 21 | 18 |
| 400 | 29 | 26 | 23 |
| 500 | 37 | 34 | 30 |
| 600 | 54 | 42 | 38 |
| 700 | 79 | 54 | 52 |
| 800 | 132 | 68 | 76 |
| 900 | 326 | 90 | 125 |
| 1000 | | 115 | 672 |
| 1100 | | 145 | |
| 1200 | | 270 | |
| 1300 | | 558 | |

From the preceding Tables it is at once apparent that, at the shorter spans, the difference of weight between the independent girders and those dependent upon some extraneous means of support is much less marked than before. Indeed, in some instances the conditions are reversed; thus, at 300 ft. span both the lattice and bowstring girders show a certain positive advantage over the arch, whilst the more economical system of the straight link retains an advantage even up to 400 ft. span.

The positions of the several types in the scale of economy would, obviously, be again shifted if the side spans were taken at a different ratio to the main span than $\frac{1}{2}$; but we believe it is unnecessary on that account to extend our investigations.

Of the numerous practical considerations and contingencies to be duly weighed and carefully estimated, before the fitness of a design for a long-span railway bridge could be satisfactorily determined, none are more important than those affecting the facility of erection. In the majority of instances scaffolding would impose a most extravagant charge on the finished structure for mere temporary works, even if the adoption of it were not practically prohibited by the necessity of maintaining the navigable channel—across which, probably, the bridge has to be thrown—free from all obstructions.

Under such circumstances there are obviously but two alternatives: the bridge must either be built at the nearest available spot, and be lifted or slidden bodily into place; or the design must be such that the structure may be built *in situ*, without adventitious support.

The latter alternative is incapable of application to either of our first four types—the independent girders. The method adopted in the instance of the Britannia

Bridge, where the tubes were built on shore, floated, and lifted into place, is an example of the former alternative applied to a box girder; and Schwedler's six-span bridge, each span of which was conveyed in a set of six-wheel wagons to the required spot, and lifted bodily into place by four hydraulic presses, illustrates the adaptation of the same system to a bowstring bridge.

The more convenient process of constructing the bridge in the position it is permanently to hold may, in one form or another, be employed for any of our type forms, not being one of the first four. Thus, it was proposed to build Mr. Fowler's Severn Bridge in successive bays projecting from each side of the two main piers, carrying on the process till the two opposing centre halves met, and formed a continuous structure.

In an arched bridge, to be erected without scaffolding, temporary ties in some form are indispensable. If it is to be lifted into place, the feet of the ribs must be tied together; and if it is to be built in position, the arched rib must be tied back to the abutments, as proposed in the first design for the Britannia Bridge.

Types 9, 10, and 11 might all be dealt with in a manner analogous to that by which the Ohio suspension bridge was successfully erected. Wire ropes were paid out over the stern of a vessel, and laid on the bed of the river, as if they were electric cables. At convenient times they were smartly hoisted up to the summit of the towers, and a slight platform was placed on them to facilitate the construction of the main cables.

If due consideration be given to these and other special conditions affecting each individual case, we think, with the assistance of the Tables already given, there will be little difficulty in ascertaining, with a considerable amount of accuracy, the most suitable form of construction for any "*long-span railway bridge.*"

# SHORT-SPAN RAILWAY BRIDGES.

# SHORT-SPAN RAILWAY BRIDGES.

THE division of railway bridges into the two classes—long and short span—is convenient for the purposes of investigation, but it must necessarily be of an arbitrary character as regards the precise limits of the two series. Under the head of "Long-span Bridges" we have considered briefly the chief characteristics of railway bridges varying from 300 ft. to 3200 ft. in span, and we now purpose to group under the head of "Short-span Bridges" all spans from the lower limit of the preceding series down to 10 ft.

The differences between the two groups of railway bridges will be of a sufficiently marked nature to justify the division. In a long-span bridge, so defined, the weight of the structure itself will constitute at all times a very important, and at the higher spans an all-important, proportion of the gross load; and, since the action of a rolling load is very different to that of a dead load of equal intensity, the general form of bridge suitable to carry a mixed load when the rolling element is of insignificant proportion may be essentially different from the design suitable for a bridge when those conditions are reversed.

Again, a bridge may be defined as of long or short span relative to the average length of train traversing it; because a train does not constitute a rolling load of uniform weight per lineal foot, and the variation in its density will be of little importance in the instance of long spans, but of vital influence in the case of short-span bridges. Thus, a passenger-train may be made up of a string of vehicles weighing less than half a ton per foot run, whilst for the 50 ft. length at the head of the train, occupied by the engine and tender, the weight may average 1¼ ton per foot, or two and a half times as much as the remainder of the train; and in the very common case of the employment of two engines and tenders the heavier load will apply of course to a length of 100 ft. of the train. In the same way a goods train may consist of a long line of trucks weighing less than ¾ ton per lineal foot, whilst the tank engine at its head may average double that amount, or 1½ ton per foot. Hence, as the wheel base of a tank engine is not unfrequently as little as one-half of the total length of engine, it is obvious at once that in short-span bridges we shall have to entertain rolling loads at least as high as 3 tons per lineal foot, although in the case of long-span bridges subject to be traversed by the same trains the average weight of the train, or little more than ¼ of the preceding amount, need be considered in the calculations.

It will, we think, be granted at once, that bridges which in the ordinary working of the traffic may be loaded throughout their entire length with engines and tenders—that is to say, bridges up to 100 ft. in span—are practically, in many essential features, short-span bridges. If due consideration be given to the influence of the heavy load at the head of the train in traversing a long-span bridge, it will be admitted also that the 300 ft. span, which was adopted in the previous portion of this work as the commencement of the long-span series and of uniform rolling loads, has not been fixed at too high a limit.

### *Rolling Load.*

In proceeding now to the consideration of short-span railway bridges, and the determination of the weight of metal which with ordinary care in design should suffice for their construction, it will be necessary in the first place to form a clear estimate of the actual practical influence of the condition already referred to—that in the mixed load upon a short-span bridge, the dead or constant load is often of insignificant proportional amount. The rolling load in the spans now under consideration will not only be relatively greater than before, but it will, as already remarked, be absolutely greater; and the effect of any alteration in the factor of safety, or in the maximum unit strain upon the metal on account of the rolling load, will obviously affect the required weight of iron in the girders in a double degree.

The special action of a rolling load upon a girder as compared with a dead load of equal intensity may be of a threefold nature:

(1.) The rolling load may impose a greater bending stress than that due to the same load in a state of rest, and this would be accompanied of course by an increased unit strain upon the metal, and an increased deflection of the girder.

(2.) The repeated applications and withdrawals of the load, with the consequent bendings of the girder, may "fatigue" the metal, and a lower unit strain may on that account be desirable.

(3.) In the case of a railway bridge with permanent way inefficiently maintained, these repeated applications of the load will be accompanied with vibrations which may have the effect of multiplying the number of bendings of the girder to an indefinite extent, and so increase the fatigue upon the metal.

Now, in the present day, if we exclude Government wagons, rude agricultural carts, and wheelbarrows, everything running upon wheels is seated upon springs, and this fact is a practical acknowledgment of the essential difference between a moving and a stationary load. When a vehicle is at rest the deflections of the springs are constant, and afford as exact a measurement of the insistent load as could be obtained by taking the wheels over a weigh bridge. If the load continued to press with the same intensity upon the wheels when in motion, springs would be useless, as their deflection would remain unaltered, and a solid block might, consequently, do their work. But experience has taught us that on the smoothest railroad the springs are at once sensitive to the slightest movement, and that they may be advantageously introduced, and come freely into play, even in the instance of the slowly-revolving turned rollers travelling on the accurately-faced roller-path of a large swing bridge. The deflection of the spring of a vehicle in motion is alternately greater and less than the mean deflection when at rest, and the irresistible inference is that the wheel will press upon the supporting medium with a maximum intensity corresponding to the greatest deflection of the spring and the weight of, and forces referable to, the wheels and axles. No better opportunity of appreciating the practical effect of this condition could be obtained than is afforded on many of the main lines of railway where a couple of trains frequently run at high speed in parallel lines and in the same direction. The observer in one train can then note the movement in the other train of the whole of the springs on one side of both engine and carriages, with the horizontal element eliminated and the vertical alone apparent. It is hardly possible under these circumstances to avoid drawing a parallel between the train and a long ship in a moderately smooth sea. The rising and falling of the several springs correspond with the undulations of the waves, the mean deflection and the mean line of flotation are nearly the same, respectively, as when the train is at rest and the ship is in perfectly smooth water; but the maximum stresses must be deduced from the maximum deflection of the springs in the one case, and from the immersion measured from the crest of the waves in the other; a very long railway carriage with a large number of wheels would have no vertical movement itself, neither would a very long ship. Expanding this deduction, we are led to the conclusion that a long-span bridge will suffer no increase of strain from the varying deflections of the springs induced by the unavoidable deviations from mathematical accuracy in the levels of the permanent way, although the rails themselves will be subject to strains, as we shall show hereafter, far greater than those due to the load at rest.

There may, however, be some other agency at work increasing the strains under a rolling load beyond that evidenced to the senses by the springs; and, as this is a question of paramount importance in the economic design of railway bridges, it is no matter for surprise that it has engaged the attention of the most eminent mathematicians and experimentalists. The results of the earlier experiments on the influence of rolling loads were certainly very alarming, as they appeared to indicate that one ton moving at 30 miles an hour was as destructive to a beam as two and a half tons at rest. Observations of the deflections of actual bridges, however, soon reassured engineers, as the deflections under speeds of even 40 or 50 miles per hour were not practically greater than those due to the same load at rest. Theoretical considerations, at first sight, might erroneously lead us to anticipate a greatly increased deflection under a rolling load, since they indicate the maximum deflection of an elastic beam under a suddenly imposed load to be double that of the final deflection after oscillations have ceased. The reason for this is obvious enough; for the resistance of the beam in deflecting to a certain extent measured in foot tons, must be equal to the work done by the load in its descent to the same extent, and as the latter is equal to the product of the load into the ultimate deflection, the former must be equal to the product of the mean resistance of the beam into the same deflection; hence the *mean* resistence of the beam must be equal to the load, and as the resistance is *nil* at the commencement of the bending, it must be equal to double the load at the termination.

But the conditions of the rolling load on a railway bridge are not analogous. The load is not sufficiently suddenly applied, and the horizontal motion and *vis viva* of the train destroy other conditions. It might at first be concluded that at least the cross girders of a bridge spaced at intervals of a yard only, and where consequently at 60 miles per hour the load would be imposed in the thirtieth part of a second, would constitute a case of suddenly imposed load and double strain, but experience does not bear out this conclusion. In an instance observed by the writer the deflection of some shallow cross girders so spaced, amounting to ⅜ of an inch under the driving wheels of the engine when at rest, was increased to ½ in. only when the engine traversed the bridge at considerable speed. In fact, in this case it was evident that the work done in deflecting the several girders was not performed, as in the case of a suddenly applied load, by a *vertical* descent of the weight at each girder, but by increased *horizontal* traction on the whole train; in the same manner as the wheel of a road vehicle passing over soft ground consolidates its path by the formation of a rut, at the expense of an additional pull, and not by an up-and-down hammering of the wheel.

One element of increased strain on the main girders of a railway bridge under a moving load is sufficiently apparent, but on investigation turns out to be insignificant in effect. The rails, if normally horizontal, will obviously be deflected below the horizontal line when the load is traversing the bridge, and the path of wheels therefore will be approximately an arc of a circle. The resulting centrifugal force will take effect upon the girder in a practical vertical direction, and in terms of the load the increased bending stress will, of course, be equal to the square of the velocity in feet per second, divided by 32 times the radius of the deflective curve in feet. In practical cases this increase, as we have before observed, will be found insignificant.

The most recent researches of mathematicians show that the passage of a train of ordinary length does not afford sufficient time for the attainment of molecular equilibrium in the girder, and that the increase of strain is not worthy of note. M. Bresse has shown that with a train of indefinite length, moving at the highest practical velocity, the strain, neglecting the effects of possible concussion, may be one-third greater than that due to the same load at rest; but that if the initial camber of the rails be four-thirds of the statical deflection, this increase, under the same reservation, will not take effect.

It is apparent, therefore, that if any very important modifications are required by reason of the rolling load in girders longer in the span than the very moderate length which may be required to eliminate the disturbances arising from the unequal deflection of the different springs of the same vehicle, it can only be to provide against the frequent bendings of the girder due to the imposition and withdrawal of the load, or the effects of shocks arising from irregularities in the roads, and other similar causes. The interposition of springs protects the ironwork to a great extent from direct impact, or difficulties would be encountered in dealing with light girders. Experiments on the effect of falling weights give very conflicting, and, in some instances, unintelligible results. It may readily be understood that a rectangular bar of any proportions, provided it be of the given weight per foot, will stand the same intensity of blow, whether it take effect on the flat side or on the edge of the bar; because the work done in fracturing the bar will be proportional to the product of the breaking weight into the ultimate deflection, which will be a constant amount for all rectangular bars of a given span and weight per foot. It is not so apparent, however, that a round bar of the same weight per foot will sustain the same blow as the rectangular bar, nor that the deflection will be proportional to the velocity of impact instead of the square of the same, yet both these facts are deduced from experiment.

Although in bridges of very moderate span the mean pressure on the springs of the carriages traversing it will differ but little from the normal pressure, and as a consequence the bending moment upon the girders will be much the same whether the load be moving or at rest, in shorter bridges, yet, where the span is small relative to the length of the carriage, a most material increase in the bending moment may have to be provided for in designing the girders. The play of the springs of an engine is a rough test of the efficiency and state of maintenance of the permanent way familiar to most locomotive superintendents. If the rails be badly maintained over a bridge, or if badly balanced engines be running on the line, the strain upon the main girders, if short, and upon the cross-girders in every case, will be increased to an extent which may at times pass unsuspectingly beyond the limits of safety. The experiments conducted by Baron Von Weber to ascertain the variations in the load upon the springs of engines gave such startling results that they demand notice in any inquiry of the present nature.

The engines employed were of two different types, but as analogous results were obtained with each, it is only necessary to take one group of experiments. The engine used in these experiments had four coupled drivers and a pair of leading wheels, and the deflections of the springs of the leading and trailing wheels were noted, to ascertain the variations in the load due to the pitching of the engine and other causes. The normal weight upon each spring of the leading wheels was from 3½ to 4 tons, and at times this weight was found to increase from 7½ to upwards of 8 tons, whilst at other times it decreased from ¼ to ½ ton only. In the case of the trailing coupled wheels the variation was also enormous, though not proportionally so great as in the more lightly-loaded leading wheels. The normal load upon each of the trailing wheels was about 5¾ tons, but at times as high a stress as upwards of 10 tons was registered, whilst at intervals the deflection of the spring would correspond to a stress of but 1½ ton.

Now, the normal weight upon the most heavily-loaded driving wheels in use on our railways is not more than 7½ tons, and great efforts are made by the employment of 8-wheel coupled engines, and other objectionable expedients, to reduce this stress. But these experiments of Baron Von Weber show that even the small light leading wheels which ordinarily pass unnoticed in our calculations, and which in the present case carried a normal load of under 4 tons, may, if the engine be badly balanced or the permanent way be not efficiently maintained, impose a greater strain upon rails and cross girders than that due to the normal pressure of our heaviest and most dreaded driving wheels.

The increase of the normal load upon both leading and trailing axles averaged about 4 tons per wheel, or from $\frac{1}{7}$ to $\frac{1}{8}$ of the total weight of the engine. The variation in the load upon the springs would no doubt be proportional to the weight of engine; hence in some of our 45-ton 6-wheel coupled tank engines, the normal load of 7½ tons per wheel might be increased at least 6 tons by the pitching of the engine, making a maximum load of 13½ tons per wheel, and even this enormous pressure might be under-estimated, because the springs would be more rigid, and the disturbance of the permanent way might be greater with such loads than with the lighter engines used in Weber's experiments.

Nor should the remarkable diminution in the pressure of the wheels, at times occurring, pass unnoticed. Apart from the influence which this condition exercises upon the maintenance of the permanent way, it will be at once apparent that it may destroy almost all the virtue of continuity so far as strain is involved, both in the rails and in the rail girders which are frequently placed between the cross girders of a bridge.

Startling as Baron Von Weber's results undoubtedly are, it can hardly fail to occur to such engineers as have not forgotten the experiments relating to the balancing of locomotives carried out by Le Chatelier a quarter of a century ago, that there may be many forces in operation tending to increase the pressure upon the rails, which would not be registered by the observations of the springs instituted by Von Weber. Anomalous though the proposition may appear, it may readily be made apparent that a diminished pressure upon the spring, in some instances, indicates an increased pressure upon the rail.

Other pressures than those due to weight of the wheels and axles are communicated directly to the rails without the intervention of springs. Thus the vertical pressure due to the obliquity of the connecting rod will be transmitted directly to the rails, as well as the centrifugal force of any unbalanced weight in the wheels and axles, or other revolving masses. A little reflection will make it evident that when the engine is travelling in a forward direction the obliquity of the connecting rod will press the driving wheels down upon the rails at each half-stroke, and at the same time lift the engine off the leading wheels by the thrust of the rod against the guide bars. The latter action will also, if no compensating beams are introduced, lift the engine to a smaller extent off the driving-wheel springs also; hence these springs will indicate a reduced pressure, although the actual pressure upon the rails will be far in excess of the normal load.

The disturbing action of the connecting rod is greatest when the wheel base is short, as in 6-wheel coupled engines, and it unfortunately happens that in this class of engine the cylinders are usually fixed at a rather steep inclination to clear the leading axle, which arrangement tends to modify the normal distribution of the load in the same direction as the obliquity of the connecting rod. The importance of the resulting modifications will be best enforced by a practical example, and for this purpose we will refer to a rather exaggerated type—the tank engines constructed for the Metropolitan Railway Company, with 20-inch cylinders by 2-feet stroke, placed at an inclination of 1 in $8\frac{1}{2}$.

The length of the connecting rod being 6 ft. 8 in., its downward thrust upon the crank pin at half-stroke will be rather more than the fraction $\frac{1}{6\frac{2}{3}} + \frac{1}{8\frac{1}{2}}$, or say $\frac{3}{11}$ of the load upon the piston. With a pressure of 130 lbs. on the square inch, that upon the 20-inch piston will be nearly 18 tons; hence the downward thrust of the rod will be $\frac{3}{11} \times 18$ tons, or nearly 5 tons, and the maximum joint vertical pressure of the two rods upon the driving axle will be $5\sqrt{2}$, or nearly 7 tons. This pressure of 7 tons will obviously be transmitted to the rails without the intervention of the springs, but the normal load upon the driving axle is not necessarily increased to the full extent of this pressure. There will be a force of 7 tons also lifting the forward end of the engine by the guide bars, and, if compensating beams are not introduced between the driving and trailing axles, this relief of weight will be shared by the leading and driving wheel springs in the respective proportions of about $\frac{2}{3}$ and $\frac{1}{3}$. Hence the total effective increased load upon the driving wheels would be $\frac{2}{3} \times 7$ tons, or $4\frac{2}{3}$ tons, and the normal load of 15 tons per axle would be diminished to $10\frac{1}{3}$ tons in the instance of the leading wheels, and increased to 19⅔ tons in that of the driving wheels, whilst the trailing wheels alone would remain unaffected by the admission of steam to the cylinders when both engines are at half-stroke.

With the compensating beams already referred to, the disturbance would be still greater, since the full pressure of 7 tons would be taken off the leading wheels and added to the drivers; hence the resultant loads upon those wheels and the trailing wheels would be 8 tons, 22 tons, and 15 tons respectively.

We have already referred to another action tending to increase the pressure upon the rails, not registered by the play of the springs—namely, the centrifugal force of unbalanced revolving masses. Very shortly after the attainment of high speeds upon railways, the oscillations arising from the rapid revolution of the cranks and their appendages became so apparent that the necessity of neutralizing the action of these masses by the insertion of opposing weights in the driving wheels was at once indicated. It was then found that even with this provision the alternating movements of the reciprocating masses induced a sinuous movement, which Fernihough proposed to check by an increase in the balance-weight—in short, by treating the reciprocating masses as merely so much addition to the revolving masses.

This arrangement will be seen to be perfectly effectual as regards the horizontal action of both reciprocating and revolving weights, but it involves an excess in the balance-weight as regards the vertical forces, and this excess must necessarily be met by the long-suffering permanent way.

Again referring to a specific example, in the engine already chosen for illustration, the weight of the reciprocating masses, exclusive of the revolving portions, is about $\frac{3}{8}$ ton for each cylinder, and if the horizontal action of this weight be neutralized, the balance-weight referred to the crank pin will obviously be $\frac{3}{8}$ ton greater than is required for the vertical action of the revolving masses, and it is equally obvious that the resulting centrifugal force will take effect upon the rail.

Since the wheels of our engine are 4 ft. diameter, at 25 miles per hour the speed of the crank pin will be $18\frac{1}{3}$ ft. per second, and the excess of centrifugal force will be : $\frac{\frac{3}{8}\text{ ton} \times 18\frac{1}{3} \times 18\frac{1}{3}}{32 \times 1} = 4$ tons.

Hence if the axis of the cylinder were in the plane of revolution of the balance-weight, and the said weight were divided between the three driving wheels on the same side of the engine—perhaps the sole advantage as regards stability peculiar to a 6-wheel coupled engine —the plus and minus variation in the load upon the driving wheel due to the excess in the balance-weight would be $1\frac{1}{3}$ tons. Under the same conditions as to axis of cylinder, we have already seen the increase due to obliquity of connecting rod and inclination of cylinder may be 5 tons; hence the total increase would be $6\frac{1}{3}$ tons, which would raise the normal load upon the driving wheel from $7\frac{1}{2}$ tons to $13\frac{5}{6}$ tons, or nearly cent. per cent.—a result practically identical with that found by Von Weber to be registered by the springs. But this increase of cent. per cent. would fortunately not be obtained in practice, since the axis of the cylinder would not be in the plane of the wheel, but a considerable distance inside it, as no one would have the temerity to build an engine of the class described with outside cylinders, whilst if we take the joint action of both cylinders, the periods of maximum effort of the two disturbing forces are not coterminous, and their effect would be less.

Sufficient, however, has been advanced to show that it is, in truth, almost impossible to over-estimate the destructive action of a heavy 6-wheel coupled tank engine with short wheel base and overhanging ends, when the permanent way is weak or badly maintained, and this fact should be kept clearly in view when proportioning the cross girders of a bridge subject to be traversed by these monsters. The successive deflections of the cross girders, if shallow, tend to set up oscillations of the engine, and unless exceptionally strong cross girders be employed, a distributing girder under each rail to ensure a moderately firm platform is a necessary condition of stability.

An instructive illustration of this fact was afforded by the recent failure of the cross girders of a railway bridge spanning the Regent's Park Canal. The cross girders referred to were of 14 ft. 6 in. span, and as their depth was from the special circumstances of the case absolutely limited to 8 in., they were necessarily placed very close together. The headway not admitting of distributing girders being placed as desirable under the rails, two longitudinal stiffening girders, 18 in. deep, were carried over the cross girders for the entire length of the bridge, and at a distance of 2 ft. 3 in. from the outer edge of each rail. Each cross girder was securely bolted to these longitudinals, and was also secured to the stiffeners of the main girders by **T** irons and gusset plates, thus the whole platform was well bound together and the several parts were mutually sustaining. The strength of the ironwork was such that, with 15 tons upon an axle, the tensile strain should not exceed 4 tons per square inch.

Notwithstanding the solidity of the structure and the low unit strain, this platform was entirely destroyed in about four years, and there can be no doubt that the work of disintegration commenced simultaneously with the adoption of a 45-ton engine upon a wheel base of but 14 ft., and an overhang at each end of more than half that amount, for the working of the traffic over this bridge.

With the results of Weber's experiments before us, a fair idea may be formed of the plunging action of such an engine when traversing the necessarily elastic platform of this bridge and the permanent way on each side of it. It is no matter for surprise that the heavy steel rails succumbed to blows which at times might be equivalent to a load of 30 tons per axle, or that a racking and gradual loosening of the component parts of the bridge platform should ensue, and its destruction in detail become merely a question of time. The dead weight of the platform itself was, of course, too small to appreciably mitigate the effect of the rolling load, and, as will hereafter be shown, the alternate downward and upward bending of the cross girders, due to the continuity of the longitudinal stiffening girders already referred to as having been secured to their upper flanges, would have a powerful influence in determining the life of the cross girders.

Notwithstanding these unfortunate conditions, the girders did not fail until upwards of 4¾ million pairs of wheels had passed over the bridge, and deflected each cross girder, of course, the same number of times. Even after the flange of each cross girder had been fractured in one or more places, an engine was, in ignorance, backed upon the bridge, and, although the bending of the platform under the great weight was so considerable that the guard in front of the driving wheels scraped the rails, there was still an ample margin of safety, as the gussets by which the cross girders were secured to the main girders still held on, and, acting as cantilevers, efficiently sustained the weight of the engine. The practical value of good mechanical attachments, and of gussets and other stiffening arrangements which do not enter into our theoretical considerations, are thus plainly manifested.

The instance of failure recorded above is, in itself, both alarming and reassuring. It is alarming inasmuch as the low unit strain upon the metal adopted in proportioning the strength of the girders in the usual way upon the basis of the maximum normal load upon a pair of wheels proved insufficient to secure the structure against the action of the exceptionally destructive engine worked over the bridge. It is reassuring in the sense that the work of destruction was gradual, and that the endurance of the cross girders was not overcome until they had sustained some five million repeated bendings.

Having thus directed attention to the vast difference which may obtain under certain conditions in practice between a rolling load and the same load in a state of rest, we may proceed to consider the effect of the frequent bendings upon the metal in girders subject to a rolling load.

At low unit strains the accumulated results of experience in every branch of construction sufficiently demonstrate the fact that the effect of repeated alternations of strain upon the endurance of the material is practically *nil.* But, on the other hand, where the unit strain at all approaches the elastic limit, the results of experience no less conclusively demonstrate the fact that the failure of the structure is merely a question of time. Mechanical engineers have arrived at their present tolerably uniform practice as regards the absolute and relative strengths of the different members of engines and machinery in general, by a tentative process. The failure of a particular portion, after a greater or less period of working, has suggested a corresponding increase in the renewed part in most instances, but in certain special instances the effect of alternating strains combined with or arising from unavoidable shocks in working, is so powerful, that engineers give up the attempt to confer permanent durability by increased strength, and accept the alternative of a limited life for this special member of the machine. Striking illustrations of this fact are afforded by forge machinery, thus, notwithstanding the enormous weight of metal employed in the construction of squeezers for blooms, the life of the squeezer arm averages but ten months, whilst that of the crank, which revolves at the rate of from 50 to 80 revolutions per minute, is but three months.

Mechanical engineers have, therefore, at all times necessarily appreciated the prejudicial action of alternating strains, but the matter has not been enforced in the same manner upon the attention of civil engineers engaged in the construction of bridges. Indeed, until the results of the exhaustive experiments carried out by Herr Wohler were made public, the absence of exact *data* afforded some excuse to engineers for the occasional underrating of the effects of repeated strains. No such an excuse can now be legitimately urged, for these experiments conclusively prove that whilst an occasional excessive strain may be risked, a frequently recurring high strain should be guarded against as no less disastrous to the structure in the long run than the imposition of the ultimate strain at once.

The higher the unit strain, the shorter will be the life of the material, though not in precise inverse proportion. Thus, in one series of experiments, the bars broke with less than half a million applications of a tensile strain, amounting to about $\frac{3}{4}$ of the ultimate resistance, whilst when the strain was reduced to about $\frac{5}{8}$ of the breaking strain, the bars sustained upwards of ten million applications of the load.

The essential difference between live and dead loads, where high unit strains are involved, was clearly exhibited in another series of experiments, where the bars broke with little more than 100,000 impositions of a "live" load, amounting to $\frac{7}{8}$ of the breaking weight, whilst when the load of equal intensity was one-half "live" and one half "dead" weight, the impositions of the live load was extended to four million times *without* fracture occurring.

In the preceding experiments the bars were well formed, and where reduced in diameter the return angles were rounded off. If these precautions are not observed, another element of danger pertains to live loads. Thus, a bar which with filled-in angles bore half a million pulls, broke with less than 40,000 when the corners were finished square. This fact bears closely upon the question of rivet holes in flanges with the consequent sudden reduction in sectional area, and indicates the advantage of employing rolled joists for bridge platforms, in instances where the cross girders are necessarily shallow and the variation in the load is great. It also indicates the advantage of dispensing with holes in flange rails, and adopting some expedient other than fang bolts for securing the rails to the sleepers.

The experiments carried out by Herr Wohler to ascertain the resistance of bars to repeated transverse stresses gave analogous results. Thus, with a *calculated* maximum tensile strain upon the fibres of 26 tons per square inch the bars bore about 170,000 repetitions of the stress, with 21 tons per square inch nearly half a million, with 17 tons upwards of four million repetitions, whilst when the calculated stress was reduced to 14 tons per square inch the bars sustained upwards of 48 million bendings without fracture.

The general tenor of the evidence adduced by Herr Wohler points to the conclusion that if a bar is subject to alternate pull and thrust—as in the instance of the central diagonals of a lattice bridge, or the parts of a continuous girder adjacent to the points of contrary flexure—the practical ultimate strength of the iron at the end of an indefinite term of years will be only about $\frac{1}{3}$ of its pristine strength; and that if the bar be subject to one class of strain only, increasing from *nil* to the maximum intensity, under the passage of a rolling load, the practical strength of the iron will be about $\frac{2}{3}$ of its original amount, whilst if the load be one-half constant and one-half variable, the resistance will be increased to $\frac{5}{6}$.

Fairbairn's experiments upon the transverse strength of cast-iron bars showed that even with a perfectly stable load, amounting to $\frac{2}{3}$ of the breaking weight, the failure of the bars was only a question of time. This would appear to indicate that Herr Wohler's results would have been still less favourable if the successive deflections to which he submitted his bars had extended over a longer series of years. It must not be forgotten, however, that the maximum stress upon the fibres of a bar, loaded to $\frac{2}{3}$ of its breaking weight, is more than $\frac{2}{3}$ of the ultimate unit resistance by reason of the law *ut tensio sic vis* not holding good at the higher strains. The same remark applies to the conclusion arrived at by the same experimenter that all the bars would fail with the repeated application of a transverse stress equal to $\frac{1}{2}$ the breaking weight.

The practical truth of Fairbairn's conclusion that a steady load of $\frac{2}{3}$ the breaking weight would, in the course of time, fracture a beam, was forcibly evidenced in the instance of the failure, in the winter of 1869, of the cast-iron girders carrying the Somerset House terrace over the dining-hall of King's College. The load upon the girders consisted solely of brick arches and gravel filling; hence the effect of live load did not enter into the consideration. The bending moment of the load, which was imposed upon the girders for many years, was equivalent to a weight of 21 tons, applied at the centre of the 19 ft. span. Both by calculation and by direct experiment the breaking weight of the girders was ascertained to be 33 tons, hence the working load was no less than $\frac{21}{33}$, or practically $\frac{2}{3}$ of the breaking weight, and the failure of the girders and consequent destruction of the fine dining-hall could afford no reasonable grounds for surprise.

Although, as already remarked, the question of time might have modified some of Herr Wohler's conclusions, the fact that at strains, *approaching the elastic limit,* a given variable load is at least as destructive as one and a half times its amount, if constant, was amply evidenced in both sets of experiments. Indeed, a much greater difference obtained in the case of a wrought-iron riveted girder, tested by Fairbairn, which, after being once broken by repeated applications of $\frac{2}{5}$ of the breaking weight, was repaired and submitted to three million applications of $\frac{1}{4}$, followed by 30,000 applications of $\frac{1}{3}$ of the breaking weight, when failure occurred.

In all the preceding instances, the load, though variable in amount, was steadily applied; had the loading been accompanied by shocks, the endurance of the iron, no doubt, would have been less.

We have now before us sufficient evidence to enable us to form a practical conclusion as to the effect of rolling loads upon girders. With low unit strains the life of a girder will be the same whether the load be live or dead, but under strains approaching the elastic limit, a load of the former class will be from $1\frac{1}{2}$ times to nearly twice as destructive as one of the latter. In practice the working strain is usually relatively low, though not sufficiently so to enable the effect of rolling load to be entirely eliminated in cases where the dead load is small.

We have already seen that in spans of even very moderate length neither the action of the springs of the vehicles traversing the bridge nor any other agency imposes an appreciably greater stress upon the girders than would be due to the same load in a state of rest. Hence in long-span bridges it is unnecessary to consider the special action of rolling load. When dealing with cross girders and short main girders it is, however, necessary to remember that the actual load to be provided for will be greater than the normal load upon the heaviest loaded wheels to an extent dependent upon the condition of the permanent way and type of engine employed, but which in ordinary cases may be taken as equal to 20 per cent. or upwards of the normal load, according to the ordinary speed of traffic over bridge.

It is also necessary to bear in mind the fact that repeated bendings may be injurious to a girder loaded with little dead weight, although it would be quite innocuous were the mixed load of live and dead weight in different proportions. In the instance of cross girders and the shorter main girders this condition will probably be sufficiently provided for, if we adopt a maximum unit strain upon the metal 20 per cent. lower than that adopted for the longer span bridges. Having provided for a maximum tensile strain of 5 tons per square inch net sectional area in our long-span bridges, we shall therefore take a strain of 4 tons per square inch for the lower spans of our series of short-span bridges, and gradually increase this strain as the proportion of dead load becomes greater, until at 200 ft. span it attains its maximum value of 5 tons per square inch.

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Having thus briefly reviewed the relative action of the rolling load in long and short span bridges, we may proceed to estimate the absolute amount of load which may be imposed upon bridges of different spans by the engines and trains now running upon our principal lines of railway.

There would appear to exist a tacit understanding amongst locomotive superintendents that whatever the type of engine may be, the weigh-bridge weight of a pair of loaded wheels must not exceed 15 tons. A few engines were constructed for the North London Railway, with 32 tons upon four coupled wheels, but in subsequent engines the wheel base was altered so as to reduce this load to the generally accepted limit.

As an example of the modern type of passenger engine for working heavy and fast trains, we may take the powerful locomotives built by Mr. Stirling to work the express traffic of the Great Northern Railway between King's Cross and Peterborough at the average speed of 47 miles per hour. Gradients of 1 in 105, and of 1 in 200, for nearly a continous length of 13 miles, have to be surmounted by the engines, with train of from 15 to 26 carriages in length. Powerful engines are consequently a necessity; the cylinders provided are 18 inches in diameter, by 2 ft. 4 in. stroke, and they are supplied with steam of 140 lbs. pressure by a boiler containing 1165 square feet of heating surface. The engines are 8-wheeled; the leading end is carried upon a 4-wheel bogie of 6 ft. 6 in. wheel base; the 8 ft. driving wheels are placed 7 ft. 9 in. behind the bogie, and the trailing wheels a further distance of 8 ft. 8 in. in the rear, hence the total wheel base of the engine is 22 ft. 11 in. The normal loads upon the several pairs of wheels in working order are 7 tons upon the front, and 8 tons upon the hind bogie wheels; 15 tons upon the driving, and 8½ tons upon the trailing wheels—total, 38½ tons, or $1\frac{3}{10}$ ton per foot run of engine length (29 ft. 7 in.). Tenders 20 ft. 7 in. long, and weighing 26½ tons loaded, are used alike for these engines and the heavy goods locomotives of the same railway; hence the total length of engine and tender is 50 ft., and the total weight 65 tons, or $1\frac{3}{10}$ ton per foot run.

As an example of powerful goods locomotive and tender we may again select an engine of the Great Northern Company's construction. The cylinder power is even greater than in the previous class of engine, as with the same stroke of 2 ft. 4 in. the diameter is 1 inch larger, or 19 inches, and the heating surface is correspondingly increased to 1352 square feet. The six coupled wheels are 5 ft. diameter, and the wheel base is 17 ft. 7 in. The load upon the leading wheels is 14 tons, upon the centre wheels 15 tons, and upon the trailing wheels 11¼ tons; the total weight of engine being 40¼ tons, or 1·43 ton per foot run of its length

(28 ft.), or 2·29 tons per foot wheel base. Since the tender used with these engines is identical with that already described, the total length of engine and tender will be 48 ft. 7 in., and the total weight 66¾ tons, or 1⅜ ton per foot run.

In a similar class of locomotive recently constructed for the London and Brighton Railway Company, with cylinders 17¼ in. diameter, by 2 ft. 2 in. stroke, and a heating surface of 1414 square feet, the weight of the engine is 35⅔ tons, or 1·34 ton per foot of engine length, (26 ft. 6 in.), or 2⅓ tons per foot of wheel base (15 ft. 3 in.).

Turning now to the far more formidable persecutors of permanent way and light cross girders—the heavy tank locomotive—we may cite as *facile princeps*, those constructed to work the St. John's Wood Railway traffic up the proposed 1 in 27 Hampstead incline. These engines combine great weight with short wheel base, and consequent great overhang at each end. The cylinders are 20 in. diameter by 2 ft. stroke, the heating surface is 1132 square feet, and the weight in working order is 45 tons, or 1½ ton per foot run of engine length (30 ft.), or no less than 3¼ tons per foot wheel base (14 ft.).

To those who have followed the previous portion of this paper, the destructive action of such an engine upon almost any substratum could form no matter for conjecture. Although the maximum load upon a pair of wheels is nominally the same in all the preceding engines, the real load in the case of the Great Northern goods engine, with its heavily loaded leading wheels, would be greater than in the instance of the express engine, and in the case of the tank engine last cited the excess of load would be still greater.

There is little probability that the effective weights of the preceding representatives of powerful engines will be exceeded in the future. The tendency is rather to adopt expedients which will reduce the stress upon permanent way, such as the better equalization of the load by compensating levers and by bogies. Mr. Fairlie's labours to attain the more perfect ideal must also bear fruit in time, if only by enforcing upon locomotive superintendents the inherent defects of many of the existing types of engines.

With these *data* as to the weights of locomotives we may now ascertain what the rolling load upon short-span bridges may amount to in practice, and it will be convenient to reduce in all cases the actual load, which of course occurs at points, to the equivalent load uniformly distributed.

Commencing with the span of 10 ft., which has been fixed as the lower limit of our series of short-span bridges, it will be at once obvious that the maximum bending moment will occur when a pair of driving wheels arrives at the centre of the span, and that, consequently, the equivalent distributed weight will be equal to double the maximum load upon a pair of wheels. With the assumed maximum normal load of 15 tons, it follows that the normal rolling load for the 10 ft. span will be 3 tons per lineal foot for each line of rails.

With a 20 ft. span the whole weight of the tank engine will rest upon the bridge, and when centrally placed the moment of the 45 tons load will be 120 foot tons; hence, dividing by $\frac{S^2}{8}$ we get $2\frac{4}{10}$ tons as the equivalent normal distributed load for each line of rails.

When the same engine is similarly placed upon a 30 ft. span, the bending moment will be nearly double the preceding, and the equivalent load will be $2\frac{1}{10}$ tons per foot.

If we admit that two such tank engines may be placed at the head of a train, they would cover a bridge of 60 ft. span, and the load might be taken as practically uniform, and at the rate of $1\frac{1}{2}$ ton per foot run.

A single Great Northern engine and tender would nearly cover the same span, and the load in the instance of the express engine would be rather more than $1\frac{1}{4}$ ton per foot run, and in that of the goods engine rather less than $1\frac{3}{8}$ ton.

Two such engines and tenders would cover a span of 100 ft., and would impose upon it a practically uniform load of $1\frac{3}{8}$ ton per foot, for each line of rails.

For spans between 100 ft. and 275 ft.—the higher limit of our series of short-span bridges—the rolling load may be considered to vary inversely as the spans, between the limits of the preceding $1\frac{3}{8}$ ton per foot run and the 1 ton per foot run which we have adopted as an ample allowance for the rolling load upon a bridge 275 ft. in span.

Tabulating the preceding results we have the *norma* rolling loads upon the spans enumerated as follow:

| Span. | Normal Rolling Load. |
|---|---|
| 10 feet. | 60 cwt. |
| 20 ,, | 48 ,, |
| 30 ,, | 42 ,, |
| 60 ,, | 30 ,, |
| 100 ,, | 27·5 ,, |
| 150 ,, | 25 ,, |
| 200 ,, | 22·5 ,, |
| 275 ,, | 20 ,, |

We have already seen that the normal rolling load upon a bridge will be increased in practice by an amount dependent upon the type of engine, construction of bridge platform, state of maintenance of permanent way, speed of trains traversing the bridge, and other considerations. It will be necessary to adopt some value for the percentage of increase, but the assumption must necessarily be arbitrary. The same remark applies to the adoption of different maximum unit strains for different proportions of rolling and dead load, for, as we have already pointed out, the experiments bearing upon the matter apply immediately to strains near to the elastic limit only. We have, however, already expressed our opinion that the variation in the maximum unit tensile strain upon this metal in railway girders may in practice be reasonably comprised within the limits of from 4 to 5 tons per square inch, and that the increase in the normal rolling load may be taken as varing from *nil* in long spans to 20 per cent. in spans subject to the load of one pair of wheels only. Starting, therefore, with these arbitrarily determined limits, the effective rolling loads, and the working strain upon the metal for the several spans, will be as follow:

| | Rolling Load per foot run Single Line. | | | Tensile strain per square inch. | | |
|---|---|---|---|---|---|---|
| Span. | Normal. | Increase. | Effective. | Standard. | Decrease. | Working. |
| | cwt. | per cent. | cwt. | tons. | per cent. | tons. |
| 10 | 60 | 20 | 72 | 5 | 20 | 4 |
| 20 | 48 | 10 | 53 | ” | 18 | $4\frac{1}{10}$ |
| 30 | 42 | 7 | 45 | ” | 16 | $4\frac{1}{5}$ |
| 60 | 30 | 3 | 31 | ” | 13 | $4\frac{1}{3}$ |
| 100 | 27·5 | nil. | 27·5 | ” | 10 | $4\frac{1}{2}$ |
| 150 | 25 | ” | 25 | ” | 5 | $4\frac{3}{4}$ |
| 200 | 22·5 | ” | 22·5 | ” | nil. | 5 |
| 275 | 20 | ” | 20 | ” | ” | 5 |

The rolling loads as given above may appear high in the instance of the shorter spans, but they are not proportionally higher than the loads taken for long spans, since an ordinary train averages only from 8 cwt. to 12 cwt. per foot run. The precise loads which should be provided for in any particular line of railway is of course a matter for the judgment of the engineer; the above Table merely represents the loads which stock at present running are capable of imposing upon bridges, and they are not based upon the improbable hypothesis of the whole bridge being loaded with engines and tenders, or other abnormal arrangements.

Having thus fixed upon the rolling load which we propose to provide for in short-span railway bridges, we must now fix in like manner upon the dead load, before an estimate of the weight of iron in such bridges can be formed.

### *Dead Load.*

Excluding for the present the main girders of the bridge, the dead load will comprise the iron or timber flooring, usually slightly covered with ballast, the permanent way, cross girders with gusset attachments to main girders, horizontal bracing at level of cross girders, and in the longer spans a second set of horizontal bracing, struts, and vertical bracing frames. In the instance of a double line of railway carried upon two main girders, the weight of the preceding elements may be taken as follows:

| Span in feet. | Dead load per foot run (exclusive of main girders). |
|---|---|
| 10 to 100 | 14 cwt. |
| 100 to 150 | 15 „ |
| 150 to 200 | 16 „ |
| 200 to 250 | 17 „ |
| 250 to 275 | 18 „ |

Where the two lines of railway are supported upon three main girders, the above loads may be reduced by 2 cwt. per foot; and where upon four girders by 4 cwt. per foot.

With both live loads and dead loads now before us, we may at once proceed to ascertain the weight of iron required in the construction of double line railway bridges from 10 ft. to 275 ft. in span—the final object of the present paper.

The type forms of bridges proposed to be considered are as follow:

### Type 1.—*Plate Girders.*

Division *a.* Two main girders, with cross girders secured to lower flanges of main girders.

„ *b.* Three main girders, with cross girders secured to lower flanges of main girders.

„ *c.* Two main girders with cross girders secured to upper flanges of main girders.

„ *d.* Three main girders with cross girders secured to upper flanges of main girders.

Division *e.* Four main girders under rails, with bracing, platform brackets, and handrail, but with no cross girders.

Type 2.—*Lattice Girders* (*ordinary construction*).

Division *a.* Two main girders with cross girders secured to lower flanges of main girders.

„ *b.* Three main girders with cross girders secured to lower flanges of main girders.

Type 3.—*Bowstring Girders* (*or Lattice Girders of special construction*).

Type 4.—*Continuous Girders.*

Division *a.* Two main plate girders with cross girders at bottom flanges.

„ *b.* Two main lattice girders (ordinary construction) with cross girders at bottom flanges.

„ *c.* Two main lattice girders (special construction) with cross girders at bottom flanges.

### *Weight of Iron in Short span Railway Bridges.*

### TYPE 1.—*Division* (*a*).

### *Cross Girders.*

It will be convenient to ascertain in the first instance the average weight of iron in the cross girders and bracing, since in bridges of this type, up to about 50 ft. in span, these members weigh as much or more than the main girders, and in the shorter spans they are necessarily all-important as regards the gross weight of iron in a bridge.

Probably the changes which have been effected in modern bridge construction are not more forcibly evidenced by any single instance than by that of the differences as regards the strength and general arrangement of the cross girders. In early railway bridges the cross girders were, apparently, considered as subject only to the same load per square foot of the platform as the main girders, and the case of the concentrated weight of a pair of driving wheels, which must come with its full intensity upon each cross girder, however close they may be spaced, was not entertained; nor indeed was it so essential to do so then, when the difference between the most heavily and lightly loaded wheels was far less marked than at present.

As an illustration of the chief points of difference in the practice of the ancients and moderns as regards cross girders, we may compare those of the Torksey Bridge with those of the bridge over the Seine at Orival, since both these bridges are fair representatives of their period.

In the former bridge the depth of the cross girders is but 14 in.; in the latter, though rather shorter in span, the depth is 40 in., or nearly three times as large. In the first instance the flanges consist simply of a pair of 2½-in. angle irons; in the latter, of a pair of 3½-in. angle iron, and a 14 in. × 11/16 in. plate. Again, the distance apart at which the cross girders are spaced is essentially different; in the old bridge they are but 2 ft. apart, in the new nearly 10 ft.

It is true that even at the present time the spacing of cross girders at 2 ft. intervals may be justifiable if care be taken to connect the whole together by stiff distributing girders, so that successive deflections of each cross girder may be superseded by a general deflection of the whole platform for at least the entire length of the wheel base of the engine. But even then, as a single girder will carry a given load with a less weight of metal than two girders, unless the load be excessive, it follows that the close arrangement can only be desirable in instances where the cross girders are necessarily very shallow, and where to get in the necessary sectional areas they would require to be of box construction if spaced far apart.

The distance apart of cross girders is governed to some extent by the position of the stiffeners in the main girders, even when of plate construction, but in the instance of lattice girders the proportions of the triangulations obviously fix the position of the cross girders in a far more arbitrary manner. In ordinary cases it may be considered as generally advantageous to space the cross girders at distances equal to about 1½ times the wheel base of two pairs of coupled driving wheels, or say from 9 to 12 ft., and to place rail girders between the cross girders under each rail. In the large bridge over the Danube, near Vienna, the cross girders are 2 ft. 7 in. in depth, and 12 ft. 6 in. apart, and the rail girders are 1 ft. 4 in. deep. If rail girders or their equivalent in the form of timber beams be omitted, the cross girders may be placed 5 ft. apart, and there can, in no ordinary case, be any justification for placing them close together, since an ordinary bridge rail and longitudinal sleeper, properly qualified to take the traffic of the railway when laid in ballast, will also be strong enough to bridge the opening between cross girders so spaced.

Considerable economy will be found to result from giving a liberal depth to the cross girders. From $\frac{1}{8}$ to $\frac{1}{7}$ of the span is not an unusual proportion for the depth in modern bridges.

The weight of the bracing will have to be considered in connection with that of the cross girders. The horizontal bracing usually comprises a single or double set of flat bar or channel iron ties intersecting at an angle of 45° and riveted to the top flange of each cross girder, and its function is to maintain the true line of the compression member of the girder, to check lateral vibrations, and to resist the force of the wind upon the bridge itself and a passing train. In old bridges bracing was frequently omitted, and the lateral stability was dependent upon the rigid attachment of the cross girders to the main girders, hence great stress at those points would occur during a gale. Continental engineers, in proportioning the strength of the horizontal bracing of their bridges, very usually take the force of the wind as limited only by the power of an ordinary passenger train to resist overturning, for they reasonably argue that if the train be blown over it cannot run over their bridge. This force of wind is far more than it is necessary to provide against in this country; indeed it is doubtful whether a greater pressure than $\frac{1}{4}$ cwt. per square foot ever takes effect upon the entire area of a reasonably sized structure, although there may be higher local impulses of pressure.

In deep girders another line of horizontal bracing may be advantageously introduced at the level of the top flanges, to preserve the line of the same under compressive strains. In shallow girders the same end is attained by the rigid attachment of the vertical stiffeners of the main girders to the cross girders, which are rendered laterally stable by the set of horizontal bracing first referred to.

The weight of the cross girders and bracing for a railway bridge to carry two lines of railway between the main girders may be taken as an average to vary from 6·7 cwt. for a 20 ft. span, to 9 cwt. for a 275 ft. span; but it will be understood that considerable modification in these weights, both of a plus and minus nature, may be effected by a variation in the depth or arrangement of the cross girders.

It only now remains to ascertain the weight of the two main girders, in order to arrive at the total weight of iron in our first class of bridges.

### *Main Girders.*

For the spans now under consideration it will be unnecessary to base our estimates of the weight of iron required in the construction of double line of railway bridges upon theoretical investigations alone, as was imperative in the instance of the unprecedented spans dealt with in the previous part of this work. Many bridges have been constructed of all spans between the limits of 10 ft. and 275 ft., and the precise *data* afforded are preferable to any theoretical investigation, as our purpose is to exhibit the average results of good practice, and not the best results attainable.

The results of ordinary practice as regards the weights of girders from 10 ft. to 200 ft. in span, loaded with from 10 cwt. per foot run, exclusive of the weight of the girder itself, to any amount required, was exhibited in the form of a diagram some years ago by the author.* Having the loads per foot run to which railway girders of different spans are subjected, the corresponding weight of any girder up to 200 ft. span can be at once obtained from the diagram. It will be necessary only to effect the modifications due to the varying tensile strains which, as previously explained, it is desirable to adopt in short-span bridges of different spans, and to correct for the increased depth which we propose to give our type bridges beyond that assumed in the diagrams. We shall thus have the weights of any of our type forms of bridges up to 200 ft. span, and in the previous part of this work we have those of bridges 300 ft. in span and upwards. There will, consequently, be no difficulty in constructing a diagram representing the weights of the two series of spans, nor in completing this diagram by a continuation of the curved line across the gap occurring between 200 and 300 ft. span.

In estimating the value of such a diagram, it must not be forgotten that the weight of a railway bridge is not a fixed arbitrary amount, but that, on the contrary, it would be a very rare coincidence if two equally skilful designers, starting with the same general *data*, produced designs requiring similar amounts of metal for their construction. Indeed, the case is not unknown of two bridges constructed from the same drawings differing 20 per cent. in weight, when one bridge has been contracted for at a lump sum and the other at a price per ton. Nor is this a matter for surprise, since a variation of 10 per cent. in the thickness of the ¼-in. web plates, or in the weight of the angle irons, cannot be detected by a casual inspection. Frequently, a comparatively small variation in the depth of the girder, or in the arrangement of the plates and covers, or stiffeners, will exercise a very appreciable effect upon the weight of the girder. Again, it is possible, by skilful design and the imposition of additional labour in the construction of a girder, to save material, whilst, on the other hand, by the effort to attain simplicity of detail, a vast amount of iron may be wasted.

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* 'Diagrams of the Weights of Girders up to 200 ft. span.' By B. Baker. London: Spon.

The diagrams upon which the following Tables are based, are intended to represent the *average* weights of girders, and the actual weight of any special existing girder might, if plotted on the diagram, fall above or below the curved line, according to the joint effect of a multitude of conflicting conditions. But the adoption of a uniform regular curved line can mislead no more than the representation of the sea level by a straight line, if it be remembered that, in both instances, waves of great and vague extent occur in reality, although ignored on the diagram.

With this preface we now give Tables exhibiting the weight of iron per foot run of the two main girders and of the entire bridge. We also give the maximum distributed load per foot run, or rather the sum of the dead load and of the maximum rolling load reduced to its equivalent in distributed load, with a view to facilitate the calculations of the sectional areas required at the different points in railway girders of the spans enumerated.

### Type 1.—*Plate Girders.*

Division (*a*). Two main girders with cross girders secured to lower flanges of main girders.

L = Maximum load in cwt. per foot run.
W′ = Weight of main girders in cwt. per foot run.
W = Gross weight of ironwork „ „

| Span in feet. | L<br>cwt. | W′<br>cwt. | W<br>cwt. |
|---|---|---|---|
| 20 | 124 | 3·4 | 10·1 |
| 25 | 114 | 3·9 | 10·6 |
| 30 | 109 | 4·4 | 11·1 |
| 35 | 103 | 5 | 11·8 |
| 40 | 98 | 5·5 | 12·3 |
| 45 | 93 | 6·1 | 12·9 |
| 50 | 90 | 6·6 | 13·5 |
| 60 | 84 | 7·7 | 14·7 |
| 70 | 82 | 8·8 | 15·9 |
| 80 | 82 | 9·9 | 17·2 |
| 90 | 82 | 11·0 | 18·3 |
| 100 | 82 | 12·1 | 19·5 |
| 120 | 82 | 14·3 | 21·9 |
| 140 | 83 | 16·5 | 24·3 |
| 160 | 84 | 18·7 | 26·7 |
| 180 | 84 | 20·9 | 29·1 |
| 200 | 85 | 23·1 | 31·5 |
| 225 | 87 | 26·4 | 35·0 |
| 250 | 90 | 30·8 | 39·6 |
| 275 | 94 | 36·2 | 45·2 |

### TYPE 1.—*Division* (*b*).

In this second division of plate girder bridges for a double line of railway, we have to consider the comparative advantages, or the reverse, of adopting one centre girder and two face girders in lieu of two face girders alone, as in the previous division.

By this arrangement a saving will be effected in the weight of the cross girders at the expense of an increase in that of the main girders. This will be true for all spans; but since the saving in the cross girders will remain almost constant, whilst the excess in the main girders will increase with the span, there must obviously be a limiting span beyond which no saving in weight of iron could be attained by the adoption of three main girders.

Saving in weight again by the three-girder system must not be taken as equivalent to saving in cost; for when the centre girder projects more than about 3 ft. above the rails, the ordinary 6 ft. space between the two lines must be increased 50 per cent. or more. It follows that the abutments must be increased in length to accommodate the wider bridge, whilst the floor of the bridge will be correspondingly increased in area. The cost of these elements alone may entirely absorb the amount saved in the ironwork; but if this were not so, there would still be other elements of increased cost to be taken into consideration, such as the greater width of formation adjacent to the bridge to admit of the splaying out of the rails to clear the centre girders, and the consequent increase in the quantities of earthwork, ballast, and even of property.

There may be a few exceptional instances where the collateral conditions may reverse the preceding conclusions, and render it practically advantageous to adopt three girders even when the weight of iron is greater than in the two-girder system. Thus in a viaduct upon screw piles it may be a matter of great convenience, both as regards erection of the work and stability, to form the piers of a row of three or four equidistant piles, which could only be done, were but two main girders employed, by the insertion of a heavy girder or bracing at the head of the piles.

It is obviously impracticable, therefore, to lay down any fixed rule as to the limiting span beyond which the three-girder division may be considered as out of the competition, but it will, we think, be rarely necessary to entertain such constructions beyond the span of 100 ft., and we have, therefore, not extended the Tables of weights of girders beyond that span.

The weight of the cross girders and bracing will in average instances vary from 4·6 cwt. at 10 ft. span to 5·2 cwt. at 100 ft., and that of the main girders of the entire ironwork and of the maximum distributed load will be as given in the following Table—the notation being as before:

### Type 1.—*Plate Girders.*

Division (*b*). Three main girders with cross girders secured to lower flanges of main girders.

| Span in feet. | L<br>cwt. | W′<br>cwt. | W.<br>cwt. |
|---|---|---|---|
| 10 | 159 | 2·7 | 7·3 |
| 15 | 136 | 3·3 | 7·9 |
| 20 | 122 | 4·0 | 8·6 |
| 25 | 113 | 4·6 | 9·3 |
| 30 | 108 | 5·3 | 10·0 |
| 35 | 102 | 6 | 10·7 |
| 40 | 97 | 6·6 | 11·4 |
| 45 | 93 | 7·3 | 12·1 |
| 50 | 89 | 8·0 | 12·9 |
| 60 | 84 | 9·3 | 14·2 |
| 70 | 82 | 10·6 | 15·6 |
| 80 | 82 | 12·0 | 17·1 |
| 90 | 83 | 13·4 | 18·5 |
| 100 | 85 | 14·9 | 20·1 |

### TYPE 1.—*Division* (*c*).

Hitherto we have dealt with the most ordinarily-encountered condition in bridge construction, of limited headway, to comply with which the main girders are necessarily placed at a sufficient distance apart to admit of the lines of rails being carried between them. In a few instances, such as that of a viaduct across a ravine or of a bridge across a river where the banks are high, or where headway is not requisite for the purposes of navigation, the main girders may be placed underneath the rails. An opportunity is thus afforded of placing the main girders closer together, and so reducing the length and weight of the cross girders. Thus, if the two main girders are spaced 16 ft. apart, the outer rail of each line of railway will bear immediately upon the main girder, hence the rolling load upon the cross girder will be reduced by one-half, and at the same time the length of span will be less by one-third. There will obviously be a saving at all spans by reason of the consequently reduced weight of the cross girders.

The gross weight of cross girders, bracing, brackets to carry platform beyond outer faces of main girders, and of the handrails, may be taken as varying from 4·8 cwt. per foot run at 20 ft. span to 6·5 cwt. at 275 ft., and the weights of main girders, total ironwork, and load as follow:

### Type 1.—*Plate Girders.*

**Division (*c*). Two main girders with cross girders secured to upper flanges of main girders, platform brackets, and handrail.**

| Span in feet. | L<br>cwt. | W'<br>cwt. | W<br>cwt. |
|---|---|---|---|
| 20 | 124 | 3·4 | 8·2 |
| 25 | 114 | 3·9 | 8·7 |
| 30 | 109 | 4·4 | 9·2 |
| 35 | 103 | 5 | 9·9 |

| Span in feet. | L cwt. | $W'$ cwt. | W cwt. |
|---|---|---|---|
| 40 | 98 | 5·5 | 10·4 |
| 45 | 93 | 6·1 | 11·0 |
| 50 | 90 | 6·6 | 11·6 |
| 60 | 84 | 7·7 | 12·8 |
| 70 | 82 | 8·8 | 14·0 |
| 89 | 82 | 9·9 | 15·1 |
| 90 | 82 | 11·0 | 16·3 |
| 100 | 82 | 12·1 | 17·5 |
| 120 | 82 | 14·3 | 19·8 |
| 140 | 83 | 16·5 | 22·1 |
| 160 | 84 | 18·7 | 24·4 |
| 180 | 84 | 20·9 | 26·8 |
| 200 | 85 | 23·1 | 29·1 |
| 225 | 87 | 26·4 | 32·5 |
| 250 | 90 | 30·8 | 37·1 |
| 275 | 94 | 36·2 | 42·7 |

### TYPE 1.—*Division* (*d*).

This division includes all instances where three main girders placed underneath the rails carry a double-line railway bridge. The remarks made under Division (*b*) as to increased width of abutment and other conflicting conditions do not apply in the present case, hence the relative advantages of this class of bridge and that last investigated will be very fairly indicated by the respective weights of iron required in their construction.

The weight of cross girders, bracing, floor brackets, and railing will vary from 2·8 cwt. per foot run at 10 ft. span to 3·4 cwt. at 100 ft., and the weight of main girders, total ironwork, and load will be as exhibited in the following Table:

### Type 1.—*Plate Girders.*

Division (*d*). Three main girders with cross girders secured to upper flanges of main girders, platform brackets, and handrail.

| Span in feet. | L<br>cwt. | W′<br>cwt. | W<br>cwt. |
|---|---|---|---|
| 10 | 159 | 2·7 | 5·5 |
| 15 | 136 | 3·3 | 6·1 |
| 20 | 122 | 4·0 | 6·8 |
| 25 | 113 | 4·6 | 7·5 |
| 30 | 108 | 5·3 | 8·2 |
| 35 | 102 | 6 | 8·9 |
| 40 | 97 | 6·6 | 9·9 |
| 45 | 93 | 7·3 | 10·3 |
| 50 | 89 | 8·0 | 11·1 |
| 60 | 84 | 9·3 | 12·4 |
| 70 | 82 | 10·6 | 13·8 |
| 80 | 82 | 12·0 | 15·2 |
| 90 | 83 | 13·4 | 16·7 |
| 100 | 85 | 14·9 | 18·3 |

### TYPE 1.—*Division* (*e*).

The final division of our first type of bridge refers to cases where four main girders—one under each rail—are employed. Obviously, no cross girders proper will be required, but angle-iron floor bearers, tying together the main girders, and extending beyond the outer faces of the latter, to carry the flooring and handrail, will have to be provided. A considerable reduction in weight will be thus attained at all spans, and in the smaller spans, where the depth of the girders admits of the trough form of construction, a much less thickness of floor or margin for construction between the underside of girders and surface of rails will be required in this than in any other system.

The weight of floor bearers, brackets, and bracing will vary from 1·5 cwt. per foot run at 10 ft. span, to 2·1 cwt. at 100 ft., and the weights of main girders, total ironwork, and load will be as follow:

### Type 1.—*Plate Girders.*

Division (*e*). Four main girders under rails, with bracing, platform, brackets, and handrail, but with no cross girders.

| Span in feet. | L<br>cwt. | W′<br>cwt. | W<br>cwt. |
|---|---|---|---|
| 10 | 158 | 3·2 | 4·7 |
| 15 | 134 | 3·9 | 5·4 |
| 20 | 121 | 4·7 | 6·2 |
| 25 | 112 | 5·5 | 7·1 |
| 30 | 107 | 6·2 | 7·8 |
| 35 | 101 | 7 | 8·6 |
| 40 | 96 | 7·8 | 9·5 |
| 45 | 92 | 8·6 | 10·3 |
| 50 | 89 | 9·3 | 11·1 |
| 60 | 83 | 11 | 12·8 |
| 70 | 82 | 12·4 | 14·3 |
| 80 | 82 | 14·1 | 16·0 |
| 90 | 83 | 15·8 | 17·8 |
| 100 | 84 | 17·7 | 19·8 |

### TYPE 2.—*Lattice Girders.*

Division (*a*).—The reduced weight of girders on the lattice principle as compared with plate girders, results not from any theoretical advantage of form, but simply because the construction of the web admits of a more accurate adjustment of the area of metal to the varying strain than can be effected in the instance of a solid plate web. This reduced quantity of metal in the web necessarily lessens the total load upon the girder; hence, indirectly, the weight of metal in the flanges is affected by the construction of the web. This condition, however, will only be of practical moment in the longer spans, as in short spans the weight of the girder itself constitutes but an insignificant proportion of the gross load.

Obviously then, if proper care be not taken in designing the web of a lattice girder the saving in weight as compared with a plate girder will be *nil.* In many existing bridges the lattice work is made up exclusively of flat bars, ill adapted to resist compressive strains, whilst at the same time the size of the bars is maintained practically constant throughout the entire span. In such instances, unless the girder is loaded with an exceptionally small load per foot run, the lattice construction can show no advantage over the plate.

Since lattice girders admit of an almost infinite variety of design as regards the arrangement of the diagonals, number of triangulations, sections of bars, and a multitude of other matters, we have not attempted to give Tables for each type of construction, but have simply taken the two broad divisions of "Lattice girders of ordinary construction" and "Lattice girders of special construction."

Under the former head we include all carefully-designed girders, with suitable sections of iron for the diagonals, and a fair proportionment of sectional area to strain at all points.

Under the latter head only those cases are concerned where every effort has been made, both by the adoption of great or varying depth and by the exact adjustment of area to strain at all points, to save material, even though it may be at the expense of a considerable increase in the labour.

The weights of such girders will therefore form the minor limit, and those already given for plate girders the major limit, of the variations which will be found to occur in practice in the weights of lattice girders, according to the degree of skill exercised in their design, or the extent to which it has been considered advisable to simplify the work of construction at the cost of material. The division now under consideration is intended to represent the mean weight of the girders comprised within the preceding limits. It will be obvious that the weight of the cross girders and bracing will be unaffected by the substitution of a lattice for a plate web, consequently it will be similar in amount to that already given in Division (*a*) of Type 1. The weights of the other elements will be as follow:

### Type 2.—*Lattice Girders* (*ordinary construction*).

Division (*a*). Two main girders with cross girders secured to lower flanges of main girders.

| Span in feet. | L<br>cwt. | W′<br>cwt. | W<br>cwt. |
|---|---|---|---|
| 20 | 124 | 3·1 | 9·8 |
| 25 | 114 | 3·6 | 10·3 |
| 30 | 108 | 4·1 | 10·8 |
| 35 | 103 | 4·5 | 11·3 |
| 40 | 97 | 5·0 | 11·8 |
| 45 | 93 | 5·4 | 12·2 |
| 50 | 89 | 5·9 | 12·8 |
| 60 | 83 | 6·8 | 13·8 |
| 70 | 81 | 7·7 | 14·8 |
| 80 | 81 | 8·6 | 15·8 |
| 90 | 81 | 9·5 | 16·8 |
| 100 | 81 | 10·4 | 17·8 |
| 120 | 80 | 12·2 | 19·8 |
| 140 | 80 | 14·0 | 21·8 |
| 160 | 81 | 15·9 | 23·9 |
| 180 | 81 | 17·7 | 25·9 |
| 200 | 82 | 19·5 | 27·9 |
| 225 | 82 | 22·1 | 30·7 |
| 250 | 85 | 25·4 | 34·2 |
| 275 | 88 | 29·5 | 38·5 |

### TYPE 2.—*Division* (*b*).

With three main girders the weight of cross girders and bracing will be the same as in a similar arrangement of plate girders, and the remarks made under Division (*b*) of Type 1, as to the difference between the real and apparent advantages of this and the preceding system of two main girders, apply with equal force. The weight per foot run of the three main girders, of the total ironwork, and of the load are tabulated below:

Type 2.—*Lattice Girders* (*ordinary construction*).

**Division (*b*). Three main girders with cross girders secured to lower flanges of main girders.**

|  | L | $W'$ | L |
|---|---|---|---|
| Span in feet. | cwt. | cwt. | cwt. |
| 10 | 159 | 2·6 | 7·2 |
| 15 | 135 | 3·1 | 7·7 |
| 20 | 122 | 3·7 | 8·3 |
| 25 | 112 | 4·2 | 8·9 |
| 30 | 107 | 4·7 | 9·4 |
| 35 | 102 | 5·3 | 10·0 |
| 40 | 96 | 5·8 | 10·6 |
| 45 | 92 | 6·3 | 11·1 |
| 50 | 88 | 6·9 | 11·8 |
| 60 | 82 | 7·9 | 12·8 |
| 70 | 80 | 9·0 | 14·0 |
| 80 | 80 | 10·1 | 15·1 |
| 90 | 80 | 11·1 | 16·2 |
| 100 | 82 | 12·2 | 17·4 |

### TYPE 3.

We have classed under one heading bowstring girders and "Lattice girders of special construction," because, in both instances, a reduction in weight has been assumed to have been effected by an expenditure of additional labour. A strict adjustment of sectional area to strain may be far more easily attained in a bowstring girder than in girders of uniform depth, because the chief members of the former girder require to be of practically uniform sectional area throughout.

In the instance of the bowstring girder, as in that of the lattice girder of special construction, a liberal depth has been assumed, and no provision has been made in the following Tables of weights for any waste of metal which may arise from unskilful design.

The cross girders and bracing will be the same as in other two-girder bridges, and the weight of the main girders, gross ironwork, and load respectively, will be as follows:

### Type 3.—*Bowstring Girders (or Lattice Girders of special construction).*

**Two main girders with cross girders secured to lower flanges of main girders.**

| | L | W' | L |
|---|---|---|---|
| Span in feet. | cwt. | cwt. | cwt. |
| 20 | 123 | 2·9 | 9·6 |
| 25 | 114 | 3·3 | 10·0 |
| 30 | 108 | 3·7 | 10·4 |
| 35 | 102 | 4·0 | 10·8 |
| 40 | 97 | 4·4 | 11·2 |
| 45 | 92 | 4·8 | 11·6 |
| 50 | 88 | 5·1 | 12·0 |
| 60 | 82 | 5·8 | 12·8 |
| 70 | 80 | 6·6 | 13·7 |
| 80 | 80 | 7·3 | 14·5 |
| 90 | 79 | 8·0 | 15·3 |
| 100 | 79 | 8·7 | 16·1 |
| 120 | 78 | 10·2 | 17·8 |
| 140 | 78 | 11·6 | 19·4 |
| 160 | 78 | 13·1 | 21·1 |
| 180 | 78 | 14·5 | 22·7 |
| 200 | 78 | 16·0 | 24·4 |

| Span in feet. | L<br>cwt. | W′<br>cwt. | W<br>cwt. |
|---|---|---|---|
| 225 | 78 | 17·8 | 26·4 |
| 250 | 79 | 20·0 | 28·8 |
| 275 | 81 | 22·8 | 31·8 |

### TYPE 4.—*Division* (*a*).

The succeeding and final type of bridge to carry a double line of railway, is that where two main continuous girders with cross girders attached to the lower flanges constitute the bridge.

Whilst in lattice girders the reduction in weight arises, as we have previously explained, from the saving in the weight of the web in the first instance, and from the reaction of this reduced load upon the weight of metal in the flanges in the second, in continuous girders these conditions are reversed. By the principle of continuity, a direct saving is effected in the flanges by reason of the sectional area required being less, and this reduction in the load affects indirectly the weight of metal required for the web.

As in the instance of the lattice girder, and for the same reasons, the reduction in weight will only be marked at the longer spans. There is, indeed, an additional reason for the small advantage which is found to result from the adoption of continuous girders for short spans; since with the small dead loads and heavy rolling loads the points of contrary flexure vary during the passage of a train so considerably that a large increase of sectional area is required both in flanges and web beyond that necessary with a larger proportion of dead load. Again, in short spans, although the maximum sectional area of the flanges will be less in a continuous than in an independent girder, that of the web will be greater; hence the principle of continuity may with certain proportions of girders entail increased weight. There are, however, frequently collateral advantages pertaining to the system, independent of the question of weight. Thus, continuous girders offer great facility for the erection of bridges with the minimum amount of temporary staging, since the superstructure can be put together upon the bank and be then hauled into position. In large girders temporary stiffeners are employed to reinforce the weak portions of the webs, and the leading ends of the girders are made temporarily stiffer by wire-rope stays. In small girders no such precautions are observed or required.

A girder may be continuous over one or more piers, and the economy of the system will, within certain limits, increase with the number of continuous spans. An infinite number of spans, with the consequent maximum economy, is a case for theory only; but a near approach to it is obtained in practice if the end spans of the continuous girder be made $\frac{8}{10}$ of the central spans. Thus with these proportions and an exclusively dead load, the difference in the sectional areas required at the centres of the end and central spans respectively would be but 2¼ per cent., whilst with a rolling load no less than seven times the dead load, the difference would still be but 5½ per cent.

The following Tables refer to such continuous girders. Where the end spans are equal to the central spans, the weight will lie between that of the more perfectly continuous girder and that of the independent girder.

Additional columns (*a* and *a'*) showing the sectional areas required at the centre and over the piers, in terms of that required at the centre of an independent girder of similar span, have been given in the following Tables with a view to facilitate the first approximation to the sectional areas.

The weight of the cross girders and bracing will be as in last Table, and that of the main girders, gross ironwork, and load respectively, will be as follows:

Type 4.—*Continuous Girders (end spans $\frac{8}{10}$ of central spans).*
Division (*a*). Two main plate girders with cross girders secured to the lower flanges of main girders.

*a* = Sectional area of flange over pier, and *a'* = sectional area at centre of span, in terms of sectional area required in an independent of the same span, and carrying the same load per foot run (L).

| Span in feet. | L<br>cwt. | W'<br>cwt. | W<br>cwt. | *a* | *a'* |
|---|---|---|---|---|---|
| 20 | 124 | 3·2 | 9·9 | ·749 | ·530 |
| 25 | 114 | 3·7 | 10·4 | ·748 | ·527 |
| 30 | 109 | 4·2 | 10·9 | ·747 | ·524 |
| 35 | 103 | 4·7 | 11·5 | ·746 | ·521 |
| 40 | 98 | 5·2 | 12·0 | ·745 | ·518 |
| 45 | 93 | 5·7 | 12·5 | ·744 | ·514 |
| 50 | 89 | 6·1 | 13·0 | ·743 | ·510 |
| 60 | 83 | 7·1 | 14·1 | ·741 | ·506 |
| 70 | 81 | 8·0 | 15·1 | ·739 | ·503 |
| 80 | 81 | 9·0 | 16·2 | ·737 | ·500 |
| 90 | 81 | 9·9 | 17·2 | ·736 | ·497 |
| 100 | 81 | 10·8 | 18·2 | ·734 | ·494 |
| 120 | 81 | 12·6 | 20·2 | ·732 | ·488 |
| 140 | 81 | 14·4 | 22·2 | ·730 | ·483 |
| 160 | 81 | 16·1 | 24·1 | ·728 | ·478 |
| 180 | 81 | 17·6 | 25·8 | ·726 | ·472 |
| 200 | 81 | 19·0 | 27·4 | ·725 | ·467 |
| 225 | 82 | 21·2 | 29·8 | ·723 | ·462 |
| 252 | 84 | 24·5 | 33·3 | ·720 | ·456 |
| 275 | 86 | 28·2 | 37·2 | ·717 | ·450 |

## TYPE 4.—*Division* (*b*).

Under this head we include, as in the same division of Type 2, all well-designed lattice girders with a fair proportion of depth to span, suitable sections of iron for the diagonals, and a reasonable adjustment of sectional area to strain. Unless these conditions be complied with, the weight of the continuous girder will approximate more closely to that of the independent girder than would appear from the following Table:

### Type 4.—*Continuous Girders.*

**Division (*b*).** **Two main lattice girders of ordinary construction, with cross girders secured to the lower flanges of main girders.**

| Span in feet. | L<br>cwt. | W'<br>cwt. | W<br>cwt. | $a$ | $a'$ |
|---|---|---|---|---|---|
| 20 | 124 | 3 | 9·7 | ·749 | ·530 |
| 25 | 114 | 3·5 | 10·2 | ·748 | ·527 |
| 30 | 109 | 4·0 | 10·7 | ·747 | ·524 |
| 35 | 103 | 4·5 | 11·3 | ·746 | ·521 |
| 40 | 97 | 5·0 | 11·8 | ·745 | ·518 |
| 45 | 93 | 5·5 | 12·3 | ·744 | ·514 |
| 50 | 89 | 5·8 | 12·7 | ·743 | ·510 |
| 60 | 83 | 6·7 | 13·7 | ·741 | ·506 |
| 70 | 80 | 7·4 | 14·5 | ·739 | ·503 |
| 80 | 80 | 8·2 | 15·4 | ·737 | ·501 |
| 90 | 80 | 9·0 | 16·3 | ·736 | ·498 |
| 100 | 79 | 9·7 | 17·1 | ·735 | ·497 |
| 120 | 79 | 11·2 | 18·8 | ·733 | ·492 |
| 140 | 78 | 13·0 | 20·8 | ·731 | ·487 |
| 160 | 78 | 14·2 | 22·2 | ·729 | ·482 |
| 180 | 78 | 15·4 | 23·6 | ·728 | ·476 |
| 200 | 78 | 16·6 | 25·0 | ·727 | ·472 |
| 225 | 79 | 18·4 | 27·0 | ·725 | ·467 |
| 250 | 80 | 20·8 | 29·6 | ·722 | ·461 |
| 275 | 82 | 23·7 | 32·7 | ·720 | ·455 |

## TYPE 4.—*Division* (*c*).

We have now arrived at the final type of bridge for a double line of railway to be considered in the present paper. The weights given in the following Table can only be safely taken to apply to bridges where every precaution has been taken to reduce the expenditure of material, notwithstanding a more than neutralizing increase in the cost of labour may be involved.

### Type 4.—*Continuous Girders.*

Division (*c*). Two main lattice girders of special construction with cross girders secured to the lower flanges of main girders.

| Span in feet. | L<br>cwt. | W′<br>cwt. | W<br>cwt. | *a* | *a′* |
|---|---|---|---|---|---|
| 20 | 123 | 2·8 | 9·5 | ·749 | ·530 |
| 25 | 113 | 3·1 | 9·8 | ·748 | ·527 |
| 30 | 108 | 3·5 | 10·2 | ·747 | ·524 |
| 35 | 102 | 3·8 | 10·6 | ·746 | ·521 |
| 40 | 96 | 4·1 | 10·9 | ·745 | ·518 |
| 45 | 92 | 4·4 | 11·2 | ·744 | ·514 |
| 50 | 88 | 4·7 | 11·6 | ·743 | ·510 |
| 60 | 82 | 5·4 | 12·4 | ·741 | ·506 |
| 70 | 79 | 6·0 | 13·1 | ·739 | ·504 |
| 80 | 79 | 6·6 | 13·8 | ·738 | ·502 |
| 90 | 78 | 7·2 | 14·5 | ·737 | ·500 |
| 100 | 78 | 7·8 | 15·2 | ·736 | ·499 |
| 120 | 77 | 8·9 | 16·5 | ·734 | ·494 |
| 140 | 76 | 10·1 | 17·9 | ·732 | ·490 |
| 160 | 76 | 11·2 | 19·2 | ·731 | ·486 |
| 180 | 75 | 12·2 | 20·4 | ·730 | ·481 |
| 200 | 75 | 13·1 | 21·5 | ·729 | ·476 |
| 225 | 74 | 14·3 | 22·9 | ·727 | ·471 |
| 250 | 75 | 15·8 | 24·6 | ·725 | ·466 |
| 275 | 76 | 17·8 | 26·8 | ·722 | ·461 |

The preceding Table is the last of the series relating to short-span railway bridges. Fewer types have been considered than in the instance of long-span bridges; but in both cases the aim has been to limit the number as much as possible, and yet at the same time include within the limits of the weights given in the Tables all types of bridges not absolutely extravagant in conception or construction. Bridges of an infinite variety of design are met with in practice, but if a due adjustment of sectional area to strain be observed, their weights will be found in some of the preceding Tables. Thus the main ribs, spandrils, horizontal girders, and bracing of a wrought-iron arched bridge weigh practically the same as the corresponding portions of a lattice two-girder bridge; but in the former case there will be a saving of about 2 cwt. per foot run in the cross girders by reason of the small depth of the arched rib admitting of intermediate ribs being introduced without increasing the total width of the bridge. A single illustration will suffice: the Victoria Bridge, constructed by Mr. Fowler, contains 22·6 cwt. of wrought iron per foot run, exclusive of ornamental ironwork; the weight of a pair of ordinary lattice girders, with cross girders for the same span of 175 ft., would by the Tables be 25·4 cwt., and deducting 2 cwt. per foot for diminished weight of cross girders, the resultant tabular weight of the wrought-iron arched bridge would be 23·4 cwt. per foot run.

Although our Tables are based upon the actual weights of existing bridges, they have not been formed by the simple process of recording such weights where they could be obtained, and interpolating the remainder. Nor could any useful Table be so constructed, for the variation in the quantity of iron in existing bridges exhibits an irregularity which is subject to no law. It might not unreasonably be supposed that, as all railway bridges have to comply with practically identical conditions, a corresponding identity would be found to exist both as regards design and the quantity of metal used in construction. Undoubtedly this should be the case; but unfortunately, from the absence of any recognized standard, each individual designer has been working more or less in the dark, without knowing whether his results were above or below the average. This alone can account for the anomalies of two bridges of similar span and type of construction entirely differing as to weight;—of lattice girders requiring a greater quantity of iron than plate girders of equal span, and of continuous girders in the same manner proving more extravagant than independent girders.

Whatever may be the reason, the fact at least is indisputable, that in a Table of the weights of existing bridges an inextricable jumble is presented, which can only be sorted and rendered of use to a designer by keeping clearly in view the theoretical conditions applicable to each design, and modifying the actual weights accordingly. It may be well to illustrate the truth of this statement by contrasting a few examples out of the mass of data as to the weight of iron in railway bridges now before us.

We may refer in the first place to a long-span bridge which has been designed and erected since the Tables of the weights of long-span bridges were published, as it affords an interesting confirmation of the results there predicted. One of the openings of the Kulemberg Viaduct measures 492 ft. in the clear, and the effective span of the girder is therefore about 500 ft. The depth of the girder at the centre is 65 ft. 8 in., at the ends 26 ft. 3 in.; hence, in form, the girder occupies a position between the ordinary lattice girder and the bowstring. By the Tables the weight of the former would be 4·8 tons per foot run of bridge, of the latter 4 tons—the mean of which, or 4·4 tons, would consequently represent the approximate weight of the Kulemberg Bridge. The actual weight is 4·3 tons, and of this a small proportion is steel.

As another example, we may cite the case of Mr. Fowler's proposed Severn Bridge, the central span of which was 600 ft. in the clear, and 610 ft. effective. The type of construction being a cantilever girder of varying economic depth, by the Table the weight should be somewhat above 3·1 tons per foot run. The actual contract weight of this bridge was 3·2 tons per foot run.

In a comparison of the weights of these two bridges it is necessary to remember that, although the weight of the larger span is the smaller per foot run, it will, from the type of construction, necessarily extend over a greater length. A just estimate can only be formed by including the weights of the two small spans adjoining the main span of the Kulemberg Viaduct, and taking the average of the three for the true weight per foot run. These spans are 187 ft. in the clear, and by taking them into consideration in the manner aforesaid we arrive at an average weight of 3·1 tons per foot run. Hence the apparent anomaly of the shorter span weighing $\frac{1}{3}$ more per foot run than the larger span vanishes; but the fact remains that the latter, though longer, is practically no more expensive; and this doubtless is due to the superior economy of the type of construction adopted in the instance of the Severn Bridge, and is nothing more than was enforced by us when treating on long-span railway bridges.

The two preceding bridges are examples of well-designed structures entirely differing as to principle, but agreeing in respect of the close approximation of weight to that which theory indicates. We may now cite a few examples to show how frequently and to what extent the reverse of this result is obtained in practice. A comparison may first be instituted between independent lattice-girder bridges—perhaps the most common type of construction for medium spans—and, unless otherwise stated, it may be assumed that the bridge for a double line of railway consists of two main girders, with cross girders attached to the lower flanges—Type 2, Division (*a*), in our classification.

Referring once more to the Kulemberg Viaduct for *data*, we have for the weight of a 264 ft. clear span, 2·4 tons per foot run of girders. The weight of the central 267 ft. span of the Boyne Viaduct as an independent girder was stated by the engineer to be equivalent to 1·5 ton per foot run. Comparing the two practically identical spans, we find an excess of weight in the former of exactly 60 per cent. The mean weight of the two is 1·85 ton, which will be found to correspond with that given in our Tables.

Compare now the 205 ft. span of the Jumna Bridge with the 202 ft. span of Mhow-ke-Multee Viaduct. The weight of the former for a double line of rails over the girders and a couple of roadways between the same is at the rate of 2·3 tons per foot run, whilst the weight of the latter for a double line of railway alone is 2·2 tons. In both instances four main girders are employed. By the Tables the weight of the bridge for railway alone would, if two main girders were substituted for the four actually used, be 1·42 ton per foot run. Adding ⅓ to this amount, to cover the extra duty the girders would have to perform in the case of a combined railway and road bridge, the tabular weight would be increased to 1·9 ton per foot run. The excess of weight in the Jumna Bridge is fully accounted for by the fact of four main girders having been employed, but in the other viaduct the excess can only be due to the uneconomical character of the design.

Descending in the scale of spans, we have the 187 ft. opening in the Kulemberg Viaduct weighing 1·5 ton per foot, which by the Tables should be 1·32 ton, or about 10 per cent. less; and we have the 172 ft. span of the Isar Bridge weighing 1·2 ton per foot, which by the Tables should be 1·25 ton, or in this instance about 5 per cent. more.

The actual weight of the lattice girder of the former viaduct corresponds with that of the plate-girder bridge in our Tables; here no economy would appear to have resulted from the lattice system in this instance.

As a more striking example of the fact that a lattice girder is not necessarily lighter than a plate girder, unless care be exercised in selecting the most suitable design for the work to be performed, we may take the case of a bridge at Battersea 120 ft. in span. The weight of the two main girders alone—which are designed to carry three lines of rails—is 1·66 ton per foot run. By the Tables the weight of two plate girders to carry a double line of railway would be ·72 ton, and if we assume that the weight would be one-half greater for three lines—which it would not—the increased amount would only be 1·08 ton; hence the lattice girders in the instance of the Battersea Bridge are at least 50 per cent. heavier than well-designed plate girders need be.

As an equally striking example of a precisely opposite character to the preceding, showing the great economy which may be effected by the adoption of the lattice system if every advantage be taken of the facilities it offers for proportioning the sectional area of the metal at all points to the strain, we may refer to the bridge over the Oder, constructed upon Schwedler's system. The span of this bridge is 108 ft., and the weight, including cross girders, ·7 ton per foot run. By the Tables the weight of a lattice bridge of "special construction" for such span would be ·84 ton per foot, which proves that too high an estimate has not been placed upon the value of skill in design. The weight of a plate-girder bridge by the Table would be 1·03 ton per foot, hence in this instance we find the plate-girder construction 50 per cent. heavier than the lattice, whilst in the last preceding example these conditions were precisely reversed, the lattice girder being 50 per cent. heavier than the plate.

Having thus exhibited the variations which occur in practice in the weight of independent lattice-girder bridges placed under apparently identical conditions, we may briefly review in the same manner a few examples of the hardly less frequently adopted system of continuous lattice-girder bridges—Type 4, Division (*b*), in our classification—unless otherwise notified.

Commencing as before with the higher spans, we may first compare the 262 ft. span of the bridge over the Danube near Vienna with the 267 ft. span of the Boyne Viaduct. The weight per foot run of the former is 1·43 ton; of the latter, 1·35 ton,—mean weight, 1·39 ton. By the Tables the weight would be 1·57 ton for a lattice bridge of ordinary construction, and 1·29 ton for one of special construction, the mean being 1·43 ton, or identical with the actual weight of the Danube Bridge.

Another confirmation of the general fairness of the Tables relating to continuous lattice-girder bridges is afforded by the bridge over the Po at Mezzana Corti, which is constructed to carry a double line of railway between the main girders and a substantial roadway over—which will add 50 per cent. to the weight of iron required for the railway alone. The span is 237 ft., and the actual weight of iron 2·1 tons per foot run. For a double line of railway the weight by the Tables would be 1·4 ton; adding 50 per cent. for the roadway, we obtain 2·1 tons—the actual weight of the structure. Again, the bridge over the Scorff, at Lorient, 210 ft. in span, weighs 1·4 ton per foot run, whilst the tabular weight is 1·3 ton, or some 7 or 8 per cent. less. On the other hand, the bridge over the Seine at Orival, 155 ft. in span, weighs 1·33 ton per foot, or but 5 per cent. less than the preceding bridge, although it is 55 ft. smaller in span. Referring again to the Boyne Bridge, it will be seen that the weight of the 267 ft.

span in that viaduct is practically identical with that of the 155 ft. span in the Seine Bridge. A more forcible illustration of the practical uselessness of unassorted facts as to the actual weights of continuous lattice girders is hardly needed, so it is only necessary now to show that similar inconsistencies occur in the weights of plate-girder bridges.

The Torksey Bridge, 130 ft. in span, weighs ·91 ton per foot. Since the girders are continuous over two spans only, the tabular weight may be taken as the mean between an independent girder and the more perfectly continuous girder of the Tables, or at the rate of 1·11 ton per foot run. The reduced weight of the actual structure is chiefly due to the lightness of the cross girders, which are of the old-fashioned type, and much below the present standard of strength.

On the other hand, it is not difficult to cite examples of equally old-fashioned bridges where the quantity of iron introduced is greater than that indicated by our Tables as necessary for the work to be performed. Take, for instance, the Brotherton Bridge, which consists of a pair of tubes 225 ft. in span, each carrying a single line of railway. The weight of iron in this bridge is 2·08 tons per foot run, and by the Tables it will appear that the two main girders and cross girders of a well-designed railway bridge of the same span would weigh 1·75 ton per foot run. Hence, in the former case the actual weight of iron is about 17 per cent. less, and in the latter about an equal amount greater, than that put down in the Table as sufficient for suitably-designed plate-girder bridges of the respective spans.

But one other pair of examples need be adduced to show that inconsistencies in the actual weights of bridges are not peculiar to the higher spans, but are equally apparent as we descend the scale. Compare the two following bridges, each 85 ft. in span. The first, over the Regent's Canal, consists of three main girders and cross girders, and its weight is 1·15 ton per foot run. The second, over the Thames in Canada, consists of two main girders and bracing for each line of railway, and its weight for a single line is ·34 ton, and for the double line ·68 ton per foot run. By the corresponding Tables the weight of the former bridge should be ·9 ton, and of the latter ·85 ton per foot run. Hence, in one instance the actual weight is 20 per cent. more, and the other 20 per cent. less, than that given in the Tables. The excess in the former arises in the cross girders, which were necessarily exceptionally shallow and costly; the decrease in the latter can only be due to the rolling load having been taken at an amount considerably below the average on existing bridges.

Sufficient evidence has now been advanced, we trust, to satisfy the reader that the Tables honestly represent the weight of iron required in the construction of railway bridges from the highest to the lowest spans. It cannot, at least, be denied that a sufficient justification has been offered of our statement that the actual weights of railway bridges, subject to identical conditions, vary to an extent which is only explicable upon the hypothesis that each designer has been working more or less in the dark, and has had no clear idea in his mind when he sat down to his work of what the weight of iron in his bridge should amount to. In the absence of a standard by which he could measure his work, it is no matter for surprise that the anomalies we have discussed are apparent on all sides, for not only may an infinite number of designs be made for the same bridge, each one worse than the former, but there are also an infinite number of ways of reducing the weight of iron in a bridge, at the expense of practical durability and solidity of construction, without reducing the theoretical strength of the structure.

Apart, however, from indifference to the weight of iron used on the one hand, and a desire to show an exceptionally economical result on the other, there are many legitimate influences in operation which may cause even the most skilful designer to exhibit inconsistencies in his successive works. The indeterminate conditions affecting a railway bridge, both in respect of the actual bending stress exerted by the rolling load, and the effect of repeated deflections upon the endurance of the ironwork, must necessarily be left unreservedly to the judgment of the engineer. He will be guided in his judgment by the facts before him, and as *data* accumulate or experience suggests he may legitimately modify his previous conclusions, and such modification will be reflected more or less in the weight of his structures. There may result either an increase or decrease in the weight of iron introduced. Thus, the present tendency is to give increased strength to railway bridges, and this is due, not so much to the adoption of a more liberal estimate of the absolute weight of the rolling load, as to the better appreciation of the disastrous action of such loads upon light bridges when the speed is great. This increased strength may, in some instances, be supplied without increasing to an equivalent extent the weight of iron in the bridge. Thus, with the facilities for obtaining long plates and angle irons now offered by manufacturers a great saving may be effected in the cover plates, and by a better arrangement of parts increased strength may not unfrequently be accompanied even by diminished weight. To attain the latter *desideratum* an entire discardal of precedent may be necessary, as in the instance of the cross girders of a bridge.

The strength of cross girders adopted in our Tables may appear to some readers inconsistent with the evidence adduced when considering the question of required strength, and it may be well to justify our conclusions.

We have seen that the increase of the normal load actually registered by the deflection of the springs in Weber's engines was nearly cent. per cent.; that the action of the connecting rod and engine and of unbalanced weights in the driving wheel may impose also a cent. per cent. increase, which would not be registered by the springs, and yet the increase upon the normal load assumed in our calculations is but 20 per cent.

Again, after showing that by Wohler's experiment a live load is, at high unit strains, at least 50 per cent. more destructive to a structure than a dead load, and that all other experimenters confirm this conclusion, we still make a reduction of 20 per cent. only in the unit strain upon our cross girders where the load is almost exclusively rolling.

The concessions made, both in respect of increased load and diminished strain, may appear unequal to the emergency if we refer only to the class of evidence given above, but there is another class of evidence which, as practical men, we cannot afford to ignore, and that is the evidence afforded by the almost innumerable present existent cross girders themselves. It cannot reasonably be omitted from the consideration, that in perhaps the larger proportion of existing railway bridges the normal load upon the driving wheels would impose a strain of 5 tons per square inch upon the cross girders. The fact that in only a perfectly insignificant number of instances has any distress been evinced by the metal, is, it is true, no conclusive proof of the propriety of the proportions adopted, but it is an ample justification for our limiting the increase of strength to the extent already indicated.

By assuming in our calculations an increase of 20 per cent. on the normal rolling load, and a decrease of an equal amount in the strain, we obtain cross girders one and a half times stronger than the preceding, and if any further proof of the sufficiency of such increase be required, it may be found in permanent way.

An ordinary iron rail upon cross sleepers constitutes, in one sense of the word, a continuous girder bridge of very small span, since the stress upon the metal can never be *less* than such conditions would involve. The stress may, however, as we have shown elsewhere,* be very much greater; indeed, if such were not the case, the lightest section of rail would be strong enough for any traffic provided the sleepers were placed sufficiently close together. We have shown that the limit in the reduction which can be effected in the weight of a rail by diminishing the distance between the sleepers, is governed by the condition that the rail must be sufficiently strong to distribute the weight of the engine upon the ballast. To comply with this condition, the strength of the rail alone in a cross sleeper road, or of the combined rail and sleeper in a longitudinal sleeper road, must be identical. This theoretical conclusion was fully confirmed in the subsequent experiments of Baron Von Weber, who summed up his results as follows:—"That increasing the number of sleepers in order to increase the carrying power of a permanent way, is theoretically and economically a wrong mode of attaining that end."

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* 'On the Strength of Beams, Columns, and Arches.' Spon.

In the work before mentioned we have shown that the bending moment upon a rail cannot safely be estimated at less than 60 inch tons, however closely the sleepers may be placed together, but that it may be greater if the space between the sleepers be excessive. Thus, with ordinary fish-joints, the bending moment upon the rail when the wheel is midway between two sleepers will be about $\frac{8}{10}$ of that due to the span. Hence, with a weight of 9 tons upon a wheel, the distance apart at which the sleepers may be spaced without exceeding the minimum bending moment of 60 inch tons will be $\frac{10 \times 60 \times 4}{8 \times 9 \text{ tons}} = 33$ in. In other words, the necessary strength of rail could not be decreased by putting the sleepers closer together than 33 in., but it would require to be increased if they were spaced at longer intervals. In practice, they could seldom be placed farther apart, as the bearing area upon the ballast would then be deficient; hence, as regards strain upon the metal, the rail may be considered as a series of imperfectly continuous girders of 33 in. span.

Now, experience informs us that an 84-lb. rail is sufficiently strong in practice for the heaviest traffic. The moment of resistance of such a rail will be about 11 in.; hence, since the moment of the bending stress is, as we have seen, 60 inch tons, the strain upon the metal will be $\frac{60}{11} = 5\frac{1}{2}$ tons per square inch. This $5\frac{1}{2}$ tons takes effect upon a solid rolled rail or girder, which is far better fitted to resist fatigue than a built-up girder; hence a reduced strain should be adopted in the latter case. In fixing the amount at 4 tons per square inch, as we have done, a sufficient provision has been made for the difference in the conditions obtaining in a rail and a cross girder. A further provision, however desirable theoretically, can hardly be considered a practical necessity, since it will be obvious that, however abnormal the stress resulting from a rolling load may be, it must first be transmitted in its full intensity to the rail before it can come upon the cross girder.

The millions of instances of short-span railway bridges afforded by the permanent way of all countries, prove, therefore, as reassuring as the results of Weber and other experimentalists proved alarming, and this is our justification for adopting a strength of cross girder smaller than that which the evidence previously adduced by us might have appeared to indicate as necessary.

We have now arrived at the end of our investigations. The farther we have advanced in our inquiry the more forcibly has the conclusion been thrust upon us that the actual weights of iron in railway bridges could only be included within the widest and most irregular limits.

Not only do modifications occur from time to time in the conditions affecting the structures we have been considering, and in the current practice as regards the principle of their construction, but the designers themselves have been subject to no acknowledged laws. Unlike the masons of old, who surprise us no less by their uniformity of practice than by their perfection of conception, our bridge builders at times exhibit an unbridled licence and an indulgence in individual vagaries fatal to all true progress of their craft. Variation, in the practice of even individual designers, is but the inevitable result of an immutable law. At times timidity is apparent, at times temerity, and the influences actuating the designer, be they subjective or objective, will be reflected in the weights of his structure, and may even be graphically represented in the form of a diagram. "As from antagonistic physical forces, so from antagonistic emotions in each man, there always results, not a medium state, but a rhythm between opposite states. The one force or tendency is not continuously counterbalanced by the other force or tendency, but now the one greatly predominates, and presently by reaction there comes a predominance of the other. That which we are shown by variations in the prices of stocks, shares, or commodities, occuring daily, weekly, or at longer intervals—that which we observe in the alternation of manias and panics, caused by irrational hopes and absurd fears—that which diagrams of these variations express by the ascents and descents of a line, now to a great height and now to an equivalent depth, we discover in all social phenomena;"* and, we may truly add, we discover in every generalization of the facts reviewed by us in treating on "Long and short-span railway bridges."

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* Herbert Spencer, 'The Study of Sociology.'

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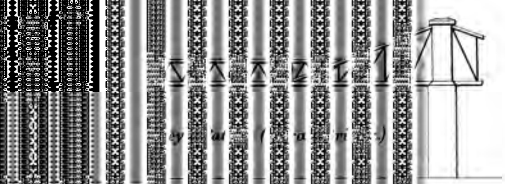

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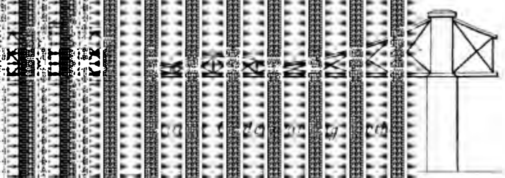

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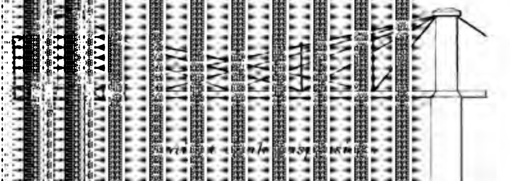

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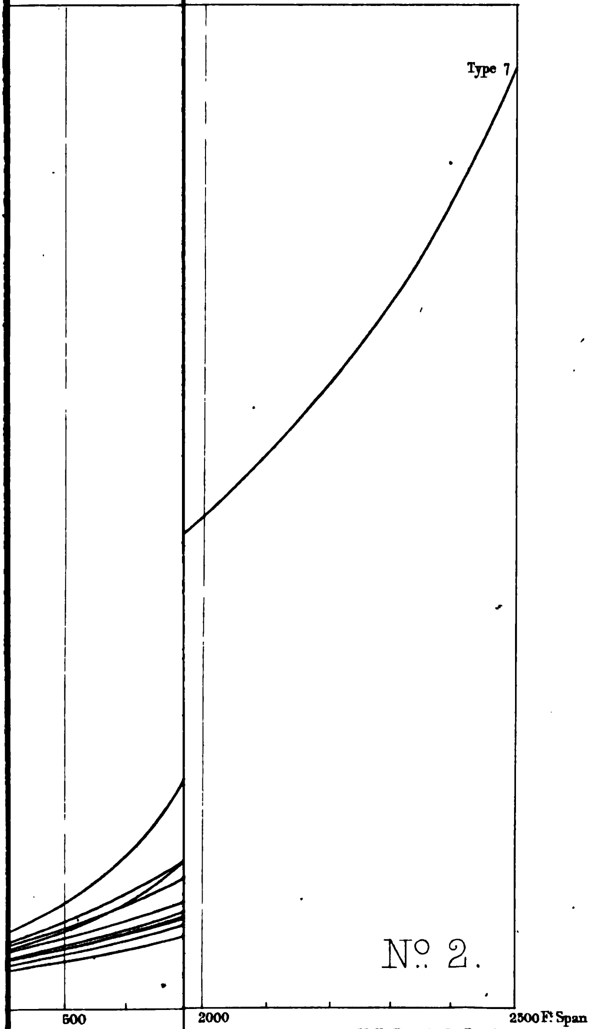

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